Boundary Polarization and the Primitive Kernel of TFPT

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Abstract

This paper isolates the primitive boundary kernel. Starting from the minimal operational seed and the one-sided boundary datum, it reconstructs the exact double, the deck involution, the Calderon polarization, the primitive admissibility complex, the primitive seam generator, the winding normalization $[u_\Sigma] = 1$, and the normalization $c_3 = 1/(8\pi)$. No Standard-Model, phenomenological, gravitational, cosmological, or E8 claim is made in this paper.

Scope box: inputs, contribution, non-claims, audit surface

Inputs from previous papers. Only the orientation map of Paper 0, if read first.
New theorem contribution. The primitive kernel

$$\mathfrak{T}^{\partial}_{\mathrm{ker}} = (\mathcal{A}, \mathcal{H}, D, J, \Gamma, \tau_{\mathrm{dbl}}, \iota_C, P_{\mathrm{prim}}, [u_\Sigma], c_3)$$

is reconstructed from the one-sided boundary datum rather than inserted later.
Not claimed here. No carrier $3 + 2$ theorem, no Standard-Model gauge group, no α, no flavor, no gravity, no cosmology, and no E8 scale grammar.
Falsification or audit surface. The paper fails if the one-sided datum does not determine the doubled datum, the Calderon polarization, the primitive Hodge selector, or the normalization of the primitive seam class.

Claim contract

Claim. $\mathfrak{S}_{\mathrm{min}} \Rightarrow \mathcal{B}_{\mathrm{min}} \Rightarrow \mathfrak{T}^{\mathrm{min}}_{\partial} \Rightarrow (\tau_{\mathrm{dbl}}, \iota_C, P_{\mathrm{prim}}, [u_\Sigma], c_3)$.
Inputs. One-sided admissible boundary datum and stated boundary-elliptic regularity.
First assumptions. Product collar, ellipticity, self-adjoint boundary realization, Calderon uniqueness, orientation normalization.
Proof status. Core theorem target for the primitive kernel.
Kill condition. Failure of the boundary datum to determine P_C, ι_C, τ_{dbl}, P_{prim}, $[u_\Sigma]$, or c_3.

Contents

1 Claim Map

The paper proves the first load-bearing arrow of the TFPT series:

$$\mathfrak{S}_{\min} \Rightarrow \mathcal{B}_{\min} \Rightarrow \mathfrak{T}_{\partial}^{\min} \Rightarrow (\tau_{\mathrm{dbl}}, \iota_{\mathrm{C}}, P_{\mathrm{prim}}, [u_{\Sigma}], c_3).$$

All later discrete, physical, and numerical structures are treated as unavailable here. The purpose is to make the boundary primitive kernel auditable on its own.

Editorial guardrail

Minimality is used only as a presentation-invariant defect filtration on essentialized admissible bordisms. It is not a preference order over desired physics. In particular, the corner count and the later $3+2$ carrier ranks are not free minimization coordinates in this paper.

2 One-Sided Boundary Datum

The starting object is

$$\mathfrak{T}_{\partial}^{\min} = (\mathcal{A}_{+}, \mathcal{H}_{+}, D_{+}, J, \Gamma, B_{\Sigma}).$$

The paper should keep only the source manuscript material needed to explain one-sided admissible data, induced boundary spectral data, and the passage to a doubled closed datum. The proof burden is that the primitive boundary structure is not chosen by hand.

3 Exact Double and Deck Involution

The exact double reconstructs

$$\mathfrak{T}^{\mathrm{cl}}_{\min} = (\mathcal{A}, \mathcal{H}, D, J, \Gamma, \tau_{\mathrm{dbl}}, \iota_C)$$

with τ_{dbl} as deck involution and ι_C as the involution induced by the Calderon polarization. This section carries the analytic interface to Calderon projectors and the primitive relative spectral dynamics from the source draft.

4 Primitive Admissibility Complex

The primitive selector is introduced before any color, determinant, family, or QFT sector:

$$P_{\mathrm{prim}} = \Pi_{\ker \Delta_{\mathrm{prim}}}.$$

The section should include the primitive admissibility differential, the primitive Hodge form, and the selector upgrade logic only as far as it is needed to prove that a later full selector can factor through P_{prim}.

5 Primitive Seam Generator

The primitive seam generator records the two normalizations that are allowed to survive this paper:

$$[u_\Sigma] = 1, \qquad c_3 = \frac{1}{8\pi}.$$

The winding class is a primitive boundary output. It is not yet a family-counting or flavor transport input; those interpretations are postponed to Papers 2 and 3.

6 Boundary Primitive Kernel

Theorem 6.1 (Primitive boundary kernel)**.** *Under the one-sided boundary hypotheses and the primitive admissibility assumptions stated above, the minimal boundary datum canonically determines*

$$\mathfrak{T}^{\partial}_{\mathrm{ker}} = (\mathcal{A}, \mathcal{H}, D, J, \Gamma, \tau_{\mathrm{dbl}}, \iota_C, P_{\mathrm{prim}}, [u_\Sigma], c_3).$$

Proof structure. The proof follows the source draft in four steps: operational seed to boundary datum; boundary datum to exact double; Calderon polarization to ι_C and P_{prim}; primitive seam normalization to $[u_\Sigma] = 1$ and $c_3 = 1/(8\pi)$. The paper must keep the representation-theoretic carrier and all observable readouts out of the argument. □

7 Audit Protocol

Audit item	Question
One-sided datum	Is every primitive object reconstructed from $\mathfrak{T}^{\min}_{\partial}$ or from standard boundary spectral data?
Calderon interface	Are regularity and projection hypotheses explicit?
Primitive selector	Does P_{prim} precede the later singlet and determinant projections?
Winding class	Is $[u_\Sigma] = 1$ fixed before any family or flavor interpretation?
Normalization	Is $c_3 = 1/(8\pi)$ derived without empirical input?

8 Main Technical Development

This section contains the main technical development assigned to this paper by the TFPT 4.5 clean split. Cross-paper background is referenced through dependency and scope boxes; extended backend material is kept in the Technical Companion.

9 Primitive core and hard carrier kernel

9.1 Primitive core and conventions

We use natural units $c = \hbar = 1$. Relative operators are formulated in Euclidean signature and compared through a declared Wick-rotation bridge when Lorentzian language is needed. Higgs fields are denoted by Φ, Hamiltonians by $\mathcal{H}$, and the cosmological Hubble rate by $H_{\text{hub}}(t)$; this avoids overloading the letter H. Newton's constant is always G_N, while G_{car} is reserved for the internal carrier stabilizer. The seam is denoted by Σ, geometric reflection by τ_{dbl}, boundary polarization by ι_C, carrier polarization by ε_{car}, and word parity by $\varepsilon(w)$.

Here $S \to \widetilde{M}$ denotes the spinor bundle on the oriented double cover, and by the same symbol we denote its restriction to

$$X_{\text{bulk}} = \widetilde{M} \setminus \Sigma.$$

When color-center notation is needed later, we use multiplicative notation

$$z(w) \in \{1, \omega, \omega^2\}, \qquad \omega = e^{2\pi i/3}.$$

Thus center neutrality means $z(w) = 1$.

$$\mathfrak{T}^{\text{kin}}_{\text{core}} := \big(\widetilde{M} \to M,\ \Sigma,\ \tau_{\text{dbl}},\ \iota_C,\ D_{\text{ref}},\ D_{\text{rel}},\ \chi_{\text{seed}}\big).$$

When a chosen almost-commutative factorization $\mathcal{H} \cong L^2(\widetilde{M}, S) \otimes \mathcal{H}_{\text{int}}$ is used, S and $\mathcal{H}_{\text{int}}$ refer to that choice and are not part of the kinematic core datum. Here "relative" always means comparison with a declared reference operator or reference-subtracted action. The canonical admissibility complex is introduced immediately below, while any Bernstein or other spectral test profile is treated only as an analytic realization choice rather than as part of the boundary datum. The technical tables for relative objects, layered closure/benchmark data, and the constants atlas are collected in Appendix 11 and Appendix [TFPT cross-reference: `app:constants`]; they are kept out of the early main-text flow so that the carrier kernel appears before the technical bookkeeping apparatus.

Remark (Boundary datum versus derived closed structure). The primitive ontology now stops one step earlier. The one-sided boundary datum $\mathfrak{T}^{\text{min}}_{\partial}$ contains only one-sided operator data together with the self-adjoint boundary operator B_Σ. Its Calderón projector defines the polarization

$$\iota_C := 2P_C - 1.$$

From that boundary polarization and the exact double one reconstructs the deck involution τ_{dbl}, the physical quotient $M = \widetilde{M}/\tau_{\text{dbl}}$, the seam $\Sigma = \text{Fix}(\tau_{\text{dbl}})$, the reference/relative operator split $(D_{\text{ref}}, D_{\text{rel}})$, the primitive seam response $\chi_{\text{seed}} = \lambda_1^+(|B_\Sigma|)^{-1}$, and the doubled closed datum $\mathfrak{T}^{\text{cl}}_{\text{min}}$. The seam is therefore no longer primitive ontology but the fixed locus of the deck involution extracted from the one-sided boundary problem.

9.2 Primitive relative spectral dynamics

The carrier theorem is not intended to float above an unspecified microscopic dynamics. The starting point is therefore the one-sided boundary datum from which the primitive layer, the derived closed datum, and the canonical admissibility projector are reconstructed before the later closure theorems are invoked.

Definition 9.1 (One-sided boundary admissibility datum)**.** The one-sided boundary admissibility datum is

$$\mathfrak{T}_{\partial}^{\min} := (\mathcal{A}_+, \mathcal{H}_+, D_+, J, \Gamma, B_\Sigma)$$

with the following built-in admissibility conditions:

(i) $Z(\mathcal{A}_+)$ is spectrally regular and its Gelfand spectrum is a smooth compact oriented manifold with boundary M_+;

(ii) B_Σ is a self-adjoint tangential boundary operator on $\Sigma := \partial M_+$ compatible with the boundary restrictions of J and Γ;

(iii) in a collar neighborhood of Σ, the Dirac operator has canonical product form

$$D_+ = \gamma_n(\partial_n + B_\Sigma);$$

(iv) the boundary collar is reflection-regular so that canonical doubling, the Calderón projector, and the admissibility complex are well posed on the same datum.

Remark (Almost-commutative realizations)*.* When a factorized realization is useful, one may choose

$$\mathcal{H} \cong L^2(\widetilde{M}, S) \otimes \mathcal{H}_{\mathrm{int}},$$

with $S \to \widetilde{M}$ the spinor bundle on the oriented double cover and $\mathcal{H}_{\mathrm{int}}$ a finite internal Hilbert space. This is an analytic realization choice rather than part of the boundary datum.

Theorem 9.2 (Doubling, deck involution, and boundary polarization)**.** *Let $\mathfrak{T}_{\partial}^{\min}$ be as above. Let*

$$\widetilde{M} := M_+ \cup_\Sigma M_+^{\mathrm{op}}$$

be the doubled manifold and let

$$\tau_{\mathrm{dbl}} : \widetilde{M} \to \widetilde{M}$$

be its deck involution. Let P_C be the Calderón projector of the APS-realized boundary problem for D_+ on Σ, and define the boundary polarization operator

$$\iota_C := 2P_C - 1$$

on the Cauchy-data Hilbert space $\mathcal{H}_\partial$.

Then:

(i) the geometric objects are

$$M = \widetilde{M}/\tau_{\mathrm{dbl}}, \qquad \Sigma = \mathrm{Fix}(\tau_{\mathrm{dbl}}), \qquad X_{\mathrm{bulk}} = \widetilde{M} \setminus \Sigma;$$

(ii) the doubled operator and its even and odd parts are

$$D = D_+ \cup_\Sigma D_+^{\mathrm{op}}, \qquad D_{\mathrm{ref}} := \frac{1}{2}\big(D + \tau_{\mathrm{dbl}}^* D \tau_{\mathrm{dbl}}^*\big), \qquad D_{\mathrm{rel}} := \frac{1}{2}\big(D - \tau_{\mathrm{dbl}}^* D \tau_{\mathrm{dbl}}^*\big);$$

(iii) on boundary Cauchy data the induced reflection $R_{\tau_{\mathrm{dbl}}}$ satisfies

$$\mathrm{Ran}(P_C) = \ker(R_{\tau_{\mathrm{dbl}}} - 1), \qquad \ker(P_C) = \ker(R_{\tau_{\mathrm{dbl}}} + 1), \qquad \iota_C = R_{\tau_{\mathrm{dbl}}}.$$

Hence the primitive scaffold

$$\mathfrak{T}^{\mathrm{kin}}_{\mathrm{prim}} := (X_{\mathrm{bulk}}, \widetilde{M}, M, \Sigma, \tau_{\mathrm{dbl}}, \iota_C, D_{\mathrm{ref}}, D_{\mathrm{rel}}, \chi_{\mathrm{seed}})$$

is derived, not assumed. The derived doubled datum may be recorded as

$$\mathfrak{T}^{\mathrm{cl}}_{\mathrm{min}} := (\mathcal{A}, \mathcal{H}, D, J, \Gamma, \tau_{\mathrm{dbl}}, \iota_C),$$

If one further chooses an almost-commutative factorization

$$\mathcal{H}_{\mathrm{prim}} \cong L^2(X_{\mathrm{bulk}}, S) \otimes \mathcal{H}_{\mathrm{int}},$$

then S and $\mathcal{H}_{\mathrm{int}}$ refer only to that chosen realization and are not additional reconstruction outputs.

Proof. Reflection regularity on an exact collar produces the doubled manifold together with the deck involution τ_{dbl}. The APS-realized boundary problem has a Calderón projector P_C whose range is the Cauchy-data space of interior solutions. On the doubled collar the deck reflection acts on Cauchy data by $R_{\tau_{\mathrm{dbl}}}$, and the solution space splits into the ± 1 eigenspaces of this reflection. Therefore

$$P_C = \frac{1 + R_{\tau_{\mathrm{dbl}}}}{2}, \qquad \iota_C = 2P_C - 1 = R_{\tau_{\mathrm{dbl}}}.$$

The formulas for D_{ref} and D_{rel} are the even and odd parts of D under the induced involution. □

Corollary 9.3 (No confusion among the three involutions)**.** *Geometric reflection is carried by τ_{dbl}, boundary polarization by ι_C, and the finite carrier split by the restricted involution*

$$\varepsilon_{\mathrm{car}} := \iota_C|_{\mathcal{H}_{\mathrm{car}}}.$$

No later theorem may identify these three operators literally; only the displayed restriction and transport maps are allowed.

The canonical admissibility projector is built in two transparent stages. The primitive stage uses only data already available from $\mathfrak{T}^{\mathrm{min}}_{\partial}$ through the derived boundary polarization ι_C, namely the seam-even selector on Cauchy data, an APS realization that makes the geometric anchor self-adjoint Fredholm, and the relative operator D_{rel}. The full stage upgrades this primitive projector by the color and determinant projectors that become available only after the carrier theorem of [TFPT cross-reference: `sec:minimal-carrier-proof`] identifies the color factor and after the compact bosonic index of [TFPT cross-reference: `sec:closure-theorems`] fixes the determinant involution. The full complex and the upgrade theorem appear once these later inputs are in place; here we close the primitive stage.

Definition 9.4 (Primitive admissibility complex and primitive projector)**.** Let the primitive Hilbert space carry only the seam factor and the gap sector accessible from D_{rel} alone:

$$\mathcal{H}_{\mathrm{prim}} = \mathcal{H}_{\Sigma} \otimes \mathcal{H}_t, \qquad \mathcal{H}^{\mathrm{ext}}_{\mathrm{prim}} := \mathcal{H}_{\mathrm{prim}} \otimes \Lambda(\eta_{\Sigma}, \eta_g),$$

with primitive seam selector

$$P_{\Sigma,+} := \frac{\mathbf{1} + \iota_C}{2}.$$

Fix $\epsilon_{\mathrm{adm}} > 0$ and let D_{prim} be the APS-realized restriction of the primitive sector operator to $\mathrm{Ran}(P_{\Sigma,+})$, controlled by D_{rel} relative to the geometric anchor. Define the primitive admissibility penalties

$$K_{\Sigma} := \sqrt{\lambda_{\Sigma}}\,\frac{\mathbf{1} - \iota_C}{2}, \qquad K_g := \sqrt{\lambda_{\mathrm{gap}}}\,\mathbf{1}_{[0,\epsilon^2_{\mathrm{adm}}/4)}\big(P_{\Sigma,+} D^{\dagger}_{\mathrm{prim}} D_{\mathrm{prim}} P_{\Sigma,+}\big),$$

the primitive admissibility differential

$$Q_{\text{prim}} := \eta_\Sigma^\dagger K_\Sigma + \eta_g^\dagger K_g,$$

and the primitive admissibility Laplacian

$$\Delta_{\text{prim}} := \{Q_{\text{prim}}, Q_{\text{prim}}^\dagger\}.$$

The primitive admissibility projector is the Hodge projector

$$P_{\text{prim}} := \Pi_{\ker \Delta_{\text{prim}}}.$$

Lemma 9.5 (Primitive nilpotency and primitive Hodge projector)**.** *On the primitive scaffold reconstructed from* $\mathfrak{T}_\partial^{\min}$*,*

$$Q_{\text{prim}}^2 = 0, \qquad \Delta_{\text{prim}} = K_\Sigma^2 + K_g^2, \qquad P_{\text{prim}} = \operatorname*{s-lim}_{\beta\to\infty} e^{-\beta(K_\Sigma^2+K_g^2)}.$$

In particular P_{prim} *depends only on* ι_C*, the APS realization, and* D_{rel}*, and is therefore reconstructed from the closed datum at the primitive level. Equivalently,*

$$P_{\text{prim}} = P_{\Sigma,+}P_{\text{prim,gap}}, \qquad P_{\text{prim,gap}} := \mathbf{1}_{[\epsilon_{\text{adm}}^2/4,\infty)}\big(P_{\Sigma,+}D_{\text{prim}}^\dagger D_{\text{prim}}P_{\Sigma,+}\big).$$

Proof sketch. The auxiliary generators $\eta_\Sigma^\dagger$ and $\eta_g^\dagger$ anticommute, so the square of Q_{prim} reduces to commutators of K_Σ and K_g. Both penalties act on commuting sectors of $\mathcal{H}_{\text{prim}}$, hence the mixed terms vanish and $\Delta_{\text{prim}} = K_\Sigma^2 + K_g^2$. The zero-temperature limit projects onto the intersection of seam-even and gapped admissible vectors, which is the factorized form recorded above. □

Definition 9.6 (Carrier polarization on the finite block)**.** Let $\mathcal{H}_{\text{car}}$ denote the finite-dimensional carrier block selected after primitive admissibility compression. Define

$$\varepsilon_{\text{car}} := \iota_C|_{\mathcal{H}_{\text{car}}}, \qquad E_- := \ker(\varepsilon_{\text{car}}+1), \qquad E_+ := \ker(\varepsilon_{\text{car}}-1).$$

Then the finite carrier decomposition is

$$E = E_- \oplus E_+.$$

Remark (Forward reference to the full admissibility complex). The full admissibility projector is obtained by upgrading P_{prim} with the color center projector P_{sing} and the determinant involution projector P_Θ, which are available only after the carrier theorem of [TFPT cross-reference: `sec:minimal-carrier-proof`] identifies the color factor $\mathcal{H}_c$ and after the compact bosonic index of [TFPT cross-reference: `sec:closure-theorems`] fixes the determinant involution Q_Θ. The full complex and the explicit upgrade are stated in [TFPT cross-reference: `def:full-admissibility-complex,thm:padm-upgrade`]; nothing in the present primitive subsection depends on those inputs.

Definition 9.7 (Primitive Euclidean action)**.** For any declared spectral test profile f, the primitive Euclidean action is the relative spectral seed

$$S_{\text{prim}}[\chi_{\text{seed}}] := \operatorname{Tr} f\left(\frac{D_{\text{rel}}}{\chi_{\text{seed}}}\right) - \operatorname{Tr} f\left(\frac{D_{\text{ref}}}{\chi_{\text{seed}}}\right) + \frac{i\pi}{2}\Delta\eta_\Sigma.$$

Gauge fixing, ghosts, and carrier-compatible bosonic fluctuations are introduced only after the hard carrier theorem and the canonical admissibility projector are in place. Whenever convenient, the compressed expression $P_{\text{adm}}AP_{\text{adm}}$ may be used as shorthand for matrix elements on the physical admissible sector, but it is not the definition of a local operator net.

Definition 9.8 (Physical admissible local net)**.** Let $\mathfrak{A}_E(\mathcal{O})$ be the Euclidean source algebra generated by local field insertions with test-function support in $\mathcal{O}$. After the later Osterwalder–Schrader reconstruction, let $\mathfrak{A}_M(\mathcal{O})$ denote the corresponding Minkowski local algebra and let $\mathcal{H}_{\mathrm{OS}}$ be the reconstructed Hilbert space. Define the physical admissible Hilbert space by

$$\mathcal{H}_{\mathrm{adm}} := \mathrm{Ran}(P_{\mathrm{adm}}) \subset \mathcal{H}_{\mathrm{OS}}.$$

Define the physical admissible local net by

$$\mathfrak{A}_{\mathrm{adm}}(\mathcal{O}) := \left\{ A|_{\mathcal{H}_{\mathrm{adm}}} \ : \ A \in \mathfrak{A}_M(\mathcal{O}),\ A\mathcal{H}_{\mathrm{adm}} \subset \mathcal{H}_{\mathrm{adm}},\ A^*\mathcal{H}_{\mathrm{adm}} \subset \mathcal{H}_{\mathrm{adm}} \right\}.$$

Thus P_{adm} acts as the selector of the physical state space, while the local net is defined by restriction to the invariant admissible subspace rather than by compression with a global projector.

Remark (Bernstein realizations as analytic choices)*.* Whenever a Bernstein representation of a spectral test profile is used, it is treated only as an analytic realization of a transfer channel already fixed by the closed datum and the canonical projector. Positivity or reflection stability therefore does not enter the main text as a primitive source of admissibility.

Theorem 9.9 (Primitive self-adjoint realization without extra analytic assumption)**.** *Let M_Λ be a finite cutoff of the primitive geometric anchor, so that M_Λ is compact with boundary. Let D_{geo} be the formally self-adjoint Dirac-type anchor and let A_∂ be an adapted boundary operator. Choose a self-adjoint D_{geo}-elliptic boundary condition B_{sa} of modified APS type:*

$$B_{\mathrm{sa}} = \begin{cases} H^{1/2}_{(-\infty,0)}(A_\partial), & \ker A_\partial = 0, \\ H^{1/2}_{(-\infty,0)}(A_\partial) \oplus L, & \ker A_\partial \neq 0, \end{cases}$$

where $L \subset \ker A_\partial$ is chosen as in the normal form for self-adjoint D-elliptic boundary conditions. Then the closed realization $D_{\mathrm{geo},B_{\mathrm{sa}}}$ is self-adjoint Fredholm with discrete spectrum. If D_{rel} is a symmetric differential perturbation of order at most one with bounded coefficients on M_Λ, then D_{rel} is infinitesimally $D_{\mathrm{geo},B_{\mathrm{sa}}}$-bounded and therefore

$$D_{\mathrm{geo},B_{\mathrm{sa}}} + D_{\mathrm{rel}}$$

is self-adjoint on the same domain. In particular the finite-cutoff relative spectral seed is well defined without an additional analytic existence hypothesis.

Proof. The existence of self-adjoint D-elliptic boundary conditions for formally self-adjoint Dirac-type operators is standard. In the zero-kernel case APS is already self-adjoint. If the adapted boundary operator has nontrivial kernel, one passes to a self-adjoint modified APS realization or, equivalently, to

$$H^{1/2}_{(-\infty,0)}(A_\partial) \oplus L,$$

with $L \subset \ker A_\partial$ chosen as in the normal form theorem for self-adjoint D-elliptic boundary conditions. The resulting realization is essentially self-adjoint on smooth sections satisfying the boundary condition. Since M_Λ is compact, coercivity at infinity is automatic and the realization is Fredholm with discrete spectrum.

For the perturbation, standard first-order elliptic regularity on a compact manifold with self-adjoint D-elliptic boundary condition gives a graph norm estimate

$$\|\psi\|_{H^1} \le C\big(\|D_{\mathrm{geo},B_{\mathrm{sa}}}\psi\| + \|\psi\|\big).$$

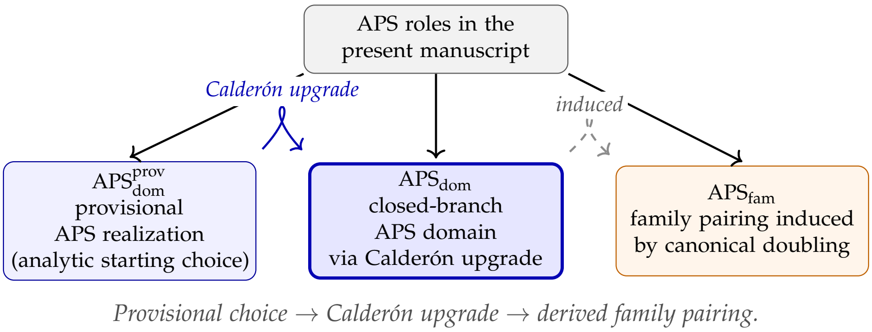

Figure 1. APS organization with Calderón upgrade and induced family pairing.

Because D_{rel} has order at most one and bounded coefficients,

$$\|D_{rel}\psi\| \le C_1\|\psi\|_{H^1} + C_0\|\psi\| \le \varepsilon\|D_{geo,B_{sa}}\psi\| + C_\varepsilon\|\psi\|$$

for every $\varepsilon > 0$. Hence D_{rel} is $D_{geo,B_{sa}}$-bounded with relative bound zero. The Kato–Rellich theorem therefore yields self-adjointness of $D_{geo,B_{sa}} + D_{rel}$ on the same domain. □

Remark (APS roles on the admissible branch). At the primitive level APS^{prov}_{dom} is the provisional elliptic boundary choice for the geometric anchor: an analytic starting representative chosen so that [TFPT cross-reference: `thm:well-posed-primitive-dynamics`] applies. After the carrier theorem and the master closure roadmap of [TFPT cross-reference: `sec:closure-theorems`] the Calderón projector of the doubled operator D_b canonically promotes APS^{prov}_{dom} to the closed-branch boundary domain APS_{dom} in the same admissible class. The family-pairing data APS_{fam} are then induced canonically and remain a derived consequence rather than a second primitive input package. The explicit upgrade statement is recorded as [TFPT cross-reference: `thm:apsdom-upgrade`] in [TFPT cross-reference: `sec:closure-theorems`].

Corollary 9.10 (Carrier-compatible Lorentzian dynamics after closure)**.** *Once the later carrier-compatible fluctuation package and the admissibility closure hypotheses of* [TFPT cross-reference: `sec:closure-theorems`] *are imposed, the resulting admissible Euclidean theory reconstructs a self-adjoint Lorentzian Hamiltonian H_{adm} and a one-parameter automorphism group*

$$\alpha_t(O) = e^{itH_{adm}} O e^{-itH_{adm}}.$$

10 Intrinsic closed branch and canonical rigid object

The closed branch is not selected by a comparison functional. At the top of the main-text theorem stack now stands a thinner operational seed layer: quasilocal Euclidean dynamics, reflection positivity, finite relative action, a nontrivial seam class, and one elliptic collar generator. That seed induces the one-sided admissible boundary datum. Once the induced datum is fixed, the remaining closure problem splits into one joint discrete admissibility theorem and one continuous retained-sector variational problem. The discrete theorem fixes carrier, family, determinant, and strong-CP data simultaneously; the continuous sector is the unique admissible critical point of one projected modular free energy on the retained branch. Their compatibility determines one intrinsic branch and therefore one canonical rigid object inside the theorem class.

10.1 Operational seed and boundary generation

Definition 10.1 (Operational seed category)**.** Let $\mathbf{Seed}^{op}$ denote the category whose objects are tuples

$$\mathfrak{S} = (\mathfrak{A}_{loc}, \tau_t, \Theta, \omega, [u_\Sigma], \mathcal{D}_{coll})$$

satisfying:

(i) $\mathfrak{A}_{\mathrm{loc}}$ is a quasilocal Euclidean observable net with positive-time reflection Θ;

(ii) ω is reflection positive and clustering on the admissible local algebra, with finite relative action with respect to the declared reference sector;

(iii) $[u_\Sigma] \neq 0$ is a nontrivial seam class;

(iv) $\mathcal{D}_{\mathrm{coll}}$ is a first-order elliptic collar generator whose positive-side restriction is reflection regular.

Morphisms are unitary intertwiners preserving the displayed structures.

Theorem 10.2 (Operational collar completion)**.** *There exists a functor*

$$\mathfrak{C}_{\mathrm{seed}} : \mathbf{Seed}^{\mathrm{op}} \longrightarrow \mathbf{SBord}^{\mathrm{adm}}$$

which assigns to every operational seed $\mathfrak{S}$ a canonically induced one-sided admissible spectral bordism

$$\mathfrak{C}_{\mathrm{seed}}(\mathfrak{S}) = \mathcal{B}(\mathfrak{S})$$

with induced boundary datum

$$\mathfrak{T}_\partial(\mathfrak{S}) = (\mathcal{A}_+, \mathcal{H}_+, D_+, J, \Gamma, B_\Sigma).$$

Proof architecture. Reflection positivity and the positive-time net reconstruct the one-sided OS Hilbert space on the collar. The first-order elliptic collar generator determines the boundary operator B_Σ, while the nontrivial seam class gives the induced spectral-flow class. The resulting one-sided tuple satisfies exactly the axioms of Definition 10.9. □

Definition 10.3 (Essentialization of admissible bordisms)**.** For every admissible bordism B write B^{triv} for the maximal direct summand on which all primitive load-bearing data vanish:

$$\boxed{\begin{array}{c}\text{seam spectral flow} = 0,\\ \text{carrier polarization acts trivially,}\\ \text{determinant degree} = 0,\\ \text{primitive Yukawa type} = 0.\end{array}} \tag{1}$$

Define the essentialized bordism by

$$B^{\mathrm{ess}} := B/B^{\mathrm{triv}}. \tag{2}$$

All minimization data below are evaluated on B^{ess} rather than on a presentation that may contain contractible spectator summands.

Definition 10.4 (Canonical defect filtration)**.** For $B \in \mathbf{SBord}^{\mathrm{adm}}$ define

$$\mathfrak{D}(B) := \big(d_0(B), d_1(B), d_2(B), d_3(B)\big) \in \mathbb{N}^4. \tag{3}$$

After replacing B with B^{ess}, set

$$d_0(B) := |\mathrm{SF}(U_\Sigma)|, \tag{4}$$

$$d_1(B) := \mathrm{rank}_{\mathrm{ess}}\, H^{\mathrm{fin}}_{\mathrm{prim}}, \tag{5}$$

$$d_2(B) := \deg^+_{\det}(B) = c_1(L_+) + c_1(L_-), \tag{6}$$

$$d_3(B) := h^{\mathrm{red}}_\Sigma(B). \tag{7}$$

Here L_+ and L_- are the determinant lines on the two boundary-polarized finite blocks before they are later named L_2 and L_3, and h_Σ^{red} is the residual boundary nullity after Calderon normal form. The order is lexicographic, but the order is not a preference weighting: each coordinate is defined only after the earlier obstruction has been minimized.

Remark (Why N_{corner} is not a minimization coordinate)*.* The four-corner structure is no longer an independent slot in the minimization functional. It is a downstream reading of the minimal nontrivial seam class together with the spin lift; schematically

$$|\,\mathrm{SF}(U_\Sigma)| = 1 \quad \Rightarrow \quad N_{\mathrm{corner}} = 4. \tag{8}$$

Objects derived from an earlier obstruction do not belong as free entries of the defect filtration.

Theorem 10.5 (Invariance of the defect lexicography)**.** *Let $\mathcal{C}$ be the category of nontrivial admissible one-sided spectral bordisms modulo unitary intertwiners and contractible stabilizations. Let*

$$\mathfrak{D} : \mathcal{C} \to \mathbb{N}^k \tag{9}$$

be the canonical defect filtration. If $F : \mathcal{C} \to \mathcal{C}'$ is an equivalence of admissible presentations and for every coordinate there is a strictly increasing map $\psi_i : \mathbb{N} \to \mathbb{N}$ with

$$d_i'(F(B)) = \psi_i(d_i(B)), \tag{10}$$

then B is a lexicographic minimizer of $\mathfrak{D}$ if and only if $F(B)$ is a lexicographic minimizer of $\mathfrak{D}'$.

Proof. Strictly increasing coordinate maps preserve and reflect the first coordinate at which two defect vectors differ. Therefore $\mathfrak{D}(B) <_{\mathrm{lex}} \mathfrak{D}(C)$ if and only if $\mathfrak{D}'(F(B)) <_{\mathrm{lex}} \mathfrak{D}'(F(C))$. Hence minimizers are preserved by equivalence. Existence follows because $\mathbb{N}^k$ is well founded under lexicographic order. Uniqueness is then reduced to classification of the final minimized stratum by the one-sided boundary datum up to unitary equivalence. □

Definition 10.6 (Operational seed defect)**.** For $\mathfrak{S} \in \mathbf{Seed}^{\mathrm{op}}$, define the pulled-back seed defect by

$$\mathfrak{D}_{\mathrm{seed}}(\mathfrak{S}) := \mathfrak{D}(\mathfrak{C}_{\mathrm{seed}}(\mathfrak{S})). \tag{11}$$

Corollary 10.7 (Minimal operational seed induces the canonical boundary datum)**.** *Let $\mathfrak{S}_{\min}$ be a defect-minimal nontrivial object of $\mathbf{Seed}^{\mathrm{op}}$ with respect to $\mathfrak{D}_{\mathrm{seed}}$. Then*

$$\mathfrak{C}_{\mathrm{seed}}(\mathfrak{S}_{\min}) = \mathcal{B}_{\min} \tag{12}$$

up to unitary equivalence, hence the canonical one-sided boundary datum $\mathfrak{T}_\partial^{\min}$ is induced by the minimal operational seed.

Proof. The seed defect is the defect filtration of the induced admissible bordism. Thus minimizing over $\mathbf{Seed}^{\mathrm{op}}$ is equivalent to minimizing the essentialized defect filtration in $\mathbf{SBord}^{\mathrm{adm}}$. The invariant-defect theorem shows that the minimizer does not depend on the chosen admissible presentation, and the induced boundary datum is $\mathfrak{T}_\partial^{\min}$. □

Axiom 10.8 (Physical operationality)**.** The physically realized world is represented by an object of $\mathbf{Seed}^{\mathrm{op}}$.

10.2 Initial admissible bordisms and induced boundary data

Definition 10.9 (Admissible spectral-bordism category)**.** Let $\mathbf{SBord}^{\mathrm{adm}}$ denote the category whose objects are tuples

$$\mathcal{B} = (\mathcal{A}_+, \mathcal{H}_+, D_+, J, \Gamma) \tag{13}$$

such that:

(i) D_+ is an elliptic Dirac-type operator on a one-sided collar with reflection-regular boundary behavior;

(ii) the retained Euclidean sector is reflection positive;

(iii) the relative action with respect to the declared reference sector is finite;

(iv) the seam class is nontrivial, equivalently $\mathrm{SF}(U_\Sigma) \neq 0$.

Morphisms are unitary intertwiners preserving the displayed one-sided data.

Theorem 10.10 (Minimal packaging inside the declared boundary class)**.** *Within the declared class of admissible one-sided boundary data encoded by nontrivial objects of* $\mathbf{SBord}^{\mathrm{adm}}$*, there exists, up to unitary equivalence, a unique essentialized defect-minimal representative*

$$\mathcal{B}_{\min}. \tag{14}$$

Its collar normal form induces a canonical one-sided boundary datum

$$\mathfrak{T}_\partial^{\min} = (\mathcal{A}_+, \mathcal{H}_+, D_+, J, \Gamma, B_\Sigma). \tag{15}$$

On the minimal branch the first defect is

$$|\,\mathrm{SF}(U_\Sigma)| = 1, \tag{16}$$

while the carrier ranks and determinant classes are not primitive minimization inputs; they are recovered later from compact Higgs selection and primitive Yukawa type. This theorem therefore shows that no extra primitive discrete datum is needed beyond the induced boundary datum.

Proof. By declaration of the admissible boundary class, the set of nonzero defect vectors

$$\{\mathfrak{D}(B) : B \in \mathbf{SBord}^{\mathrm{adm}},\ \mathrm{SF}(U_\Sigma) \neq 0\} \subset \mathbb{N}^4 \tag{17}$$

is nonempty. Well-foundedness gives a minimizer. Essentialization removes contractible spectator summands, so rank minimization cannot be changed by adding an empty internal factor. The first coordinate is the nontrivial seam obstruction; hence the minimal positive value is $|\,\mathrm{SF}(U_\Sigma)| = 1$. The Dirac collar normal form gives

$$D_+ = \gamma_n(\partial_n + B_\Sigma) \tag{18}$$

with self-adjoint adapted boundary operator B_Σ. Reflection positivity on the retained sector gives the exact double and Calderon polarization used by the primitive reconstruction. The invariance theorem makes the minimized datum presentation independent, and the boundary reconstruction theorem identifies the induced one-sided datum up to unitary equivalence. □

Remark (Formal start of the theorem chain)*.* The displayed reconstruction chains later in the manuscript may be written either from the minimal operational seed or, once the seed-to-boundary passage has been fixed, formally from $\mathfrak{T}_\partial^{\min}$, because every downstream theorem is phrased in boundary-data language. The present theorem explains why, within the induced admissible boundary class, that datum needs no second independent primitive discrete layer.

Remark (Historical seven-sector presentation). The older seven-sector closure map is retained only as a mnemonic decomposition of the eventual branch coordinates. It is no longer the proving engine of the paper. The actual closure argument is the primitive-completion argument of Section 9.1: one boundary primitive kernel fixes one joint discrete datum, one projected modular free energy fixes the continuous sectors, and the intrinsic branch is their singleton fiber product over the same upstream invariants.

Definition 10.11 (TFPT core class)**.** Let $\mathbf{TFPT}^{\mathrm{core}}$ denote the class of tuples

$$\mathfrak{T}_{\mathrm{core}} := (\mathcal{A}, \mathcal{H}, D, J, \Gamma, \tau_{\mathrm{dbl}}, \iota_{\mathrm{C}}, P_{\mathrm{prim}})$$

where P_{prim} is reconstructed from the primitive admissibility complex. Morphisms are unitary isomorphisms intertwining the displayed core data. No family geometry, no family local system class, no determinant classes, no transport pole, no strong-CP datum, and no cosmology datum are part of this definition.

10.3 Primitive completion from the boundary kernel

Definition 10.12 (Boundary primitive kernel)**.** Let

$$\mathfrak{T}^{\partial}_{\mathrm{ker}} := (\mathcal{A}, \mathcal{H}, D, J, \Gamma, \tau_{\mathrm{dbl}}, \iota_{\mathrm{C}}, P_{\mathrm{prim}}, [u_\Sigma], c_3)$$

be the boundary primitive kernel attached to an admissible one-sided boundary datum. Here

$$[u_\Sigma] = 1, \qquad c_3 = \frac{1}{8\pi},$$

are fixed by the primitive seam generator theorem. No carrier split, no hypercharge operator, no family geometry, no determinant class, no transport pole, no effective strong angle, and no vacuum datum are part of this definition.

11 Technical conventions and data layers

This appendix collects the technical bookkeeping that supports the boundary-to-carrier presentation but would slow the early main-text flow if placed before the carrier theorem.

11.1 Relative objects

The adjective "relative" is used in one fixed sense throughout the paper: every operator or action is compared to a declared reference object.

11.2 Layered datum and admissibility

Construction 11.1 (Layered datum)**.** The technical bookkeeping behind the manuscript uses five levels:

$$\mathfrak{T}^{\mathrm{kin}}_{\mathrm{core}} = (\widetilde{M} \to M,\ \Sigma,\ \tau_{\mathrm{dbl}},\ \iota_{\mathrm{C}},\ D_{\mathrm{ref}},\ D_{\mathrm{rel}},\ f,\ \chi_{\mathrm{seed}}),$$

$$\mathfrak{T}^{\partial}_{\mathrm{ker}} := ((\mathcal{A}, \mathcal{H}, D, J, \Gamma, \tau_{\mathrm{dbl}}, \iota_{\mathrm{C}}, P_{\mathrm{prim}}),\ [u_\Sigma],\ c_3),$$

$$\mathfrak{T}_{\mathrm{ker}} = ((\mathcal{A}, \mathcal{H}, D, J, \Gamma, \iota_{\mathrm{C}}, P_{\mathrm{prim}}, P_{\mathrm{adm}}),\ E_3 \oplus E_2,\ Y,\ [u_\Sigma],\ c_3),$$

$$\mathfrak{T}_{\mathrm{bridge}} = (\mathcal{R}_{\mathrm{SM}},\ \alpha(0),\ M_Z,\ \{m_c, m_b, m_\tau, m_t\}),$$

$$\mathfrak{T}_{\mathrm{comp}} = \begin{pmatrix} (\mathsf{Adm}, K_{\mathrm{adm}}, P_{\mathrm{adm}}),\ (\chi_{\mathrm{geo}}, \mathcal{B}_{\mathrm{rel}}, \mathbb{A}_\Sigma),\ (F, T), \\ (\mathcal{U}_6, D_y, \mathcal{Y}^{(\varepsilon)}_y, \epsilon_f),\ (Z_{\mathrm{rel}}, \Theta, \mathcal{H}_{\mathrm{rel}}, \mathcal{V}_{\mathrm{adm}}), \\ (\mathcal{A}_{\mathrm{rec}}, \mathcal{A}_{\mathrm{obs}}, \mathrm{Pred}) \end{pmatrix}.$$

Here $\mathfrak{T}^{\partial}_{\mathrm{ker}}$ is the primitive boundary kernel, while $\mathfrak{T}_{\mathrm{ker}}$ is the derived theorem-level post-carrier kernel reconstructed from the joint discrete datum and the full selector.

Relative object	Definition or schematic form	Role
$D_{\rm rel}$	$D - D_{\rm ref}$ or a specified relative pair $(D, D_{\rm ref})$	fermionic comparison operator
$\Gamma_{\rm grav}$	$-6\chi^2_{\rm geo}\mathcal{E}_D + \Gamma^{(4)}_D$ with $\mathcal{E}_D = \mathrm{Res}_{s=1}\,\mathcal{Z}_{\rm rel}$ and $\Gamma^{(4)}_D = \mathrm{FP}_{s=0}\,\mathcal{Z}_{\rm rel}$	canonical spectral Einstein functional from relative residues
$\Gamma_{\rm rel}$	admissible relative action preserved by the sheet antiunitary involution	strong-CP and reflection-symmetry package
$\mathrm{Ind}_{\rm rel}$	APS-type index of a relative pair or superconnection package	carrier and bosonic counting
$Z_{\rm rel}$	$Z_{\rm adm}[J, \eta, \bar{\eta}]/Z_{\rm ref}[0, 0, 0]$	normalized admissible generating functional
$P_{\rm adm}$	$\text{s-lim}_{\beta\to\infty}\, e^{-\beta K_{\rm adm}}$; factorized form when the elementary projectors commute	operative admissibility selector on the closed branch
$\langle W(C)\rangle_{\rm rel}$	Wilson observable after subtraction of the declared reference sector	confinement diagnostics
$\mathcal{H}_{\rm rel}$	relative Hamiltonian used in admissible-sector minimization	optional record extension

Table 1. Conventions on relative objects.

Thus $\mathfrak{T}^{\rm kin}_{\rm core}$ is the appendix bookkeeping rewrite of the primitive kinematic scaffold, while Adm first enters the layered datum only at closure level through $(\mathsf{Adm}, K_{\rm adm}, P_{\rm adm})$. When an almost-commutative factorization $\mathcal{H} \cong L^2(\widetilde{M}, S) \otimes \mathcal{H}_{\rm int}$ is chosen, S and $\mathcal{H}_{\rm int}$ refer to that factorization and are not additional entries in $\mathfrak{T}^{\rm kin}_{\rm core}$.

Optional horizon and far-downstream E_8 data are kept outside this compact layered datum and are introduced only in the dedicated appendix-level extension.

11.3 Symbol guide

The symbol guide is split into two tables to keep theorem-level objects lexically separate from appendix-only comparison and readout objects. Table 3 contains only symbols that participate in the main-text theorem chain; Table 4 contains only symbols that live in the appendix-level comparison and readout layer.

Definition 11.2 (Admissibility predicate and selector)**.** In the present paper Adm denotes a predicate on relative sectors, and $P_{\rm adm}$ denotes its operator realization on the retained closed branch. We write $\mathsf{Adm}(X) = 1$ precisely when:

(1) X lies in the declared domain of the relative operator pair,

(2) the associated relative action or index density is finite after reference subtraction,

(3) the required seam-evenness condition is satisfied,

(4) colored sectors satisfy center neutrality when a hadronic interpretation is intended,

(5) gap-stable transport positivity and the declared closure conditions hold in those sectors used for observable or transport statements.

On those sectors, the selector language and the predicate language are identified by

$$\mathsf{Adm}(X) = 1 \quad \Longleftrightarrow \quad P_{\rm adm}X = X.$$

Remark (Operator realization of admissibility)*.* If one wants an operator language rather than a predicate language, the natural positive generator is $K_{\rm adm}$ and its commuting-factor realization

Level	Objects	Claim role	Function in the present paper
Core datum	$\mathfrak{T}_{\partial}^{\min}$, $\widetilde{M} \to M$, Σ, τ_{dbl}, ι_C, D_{ref}, D_{rel}, χ_{seed}	Boundary datum / theorem	one-sided input plus minimal kinematic scaffold before carrier data and admissibility closure; S and $\mathcal{H}_{\text{int}}$ refer only to a chosen realization
Post-carrier primitive kernel	$(\mathcal{A}, \mathcal{H}, D, J, \Gamma, \iota_C, P_{\text{prim}}, P_{\text{adm}})$, $E_3 \oplus E_2$, Y, $[u_\Sigma]$, c_3	Theorem	primitive closure kernel carrying the admissibility, carrier, and seam-normalization data before the discrete/continuous branch split
Closure data	$(\mathsf{Adm}, K_{\text{adm}}, P_{\text{adm}})$, (F, T), $(D_{\text{geo}}, \chi_{\text{geo}}, \mathcal{B}_{\text{rel}}, \mathbb{A}_\Sigma)$, $(U_6, D_y, \mathcal{Y}_y^{(\varepsilon)}, \epsilon_f)$, $(Z_{\text{rel}}, \Theta, \mathcal{H}_{\text{rel}}, \mathcal{V}_{\text{adm}})$, $(\mathcal{A}_{\text{rec}}, \mathcal{A}_{\text{obs}}, \mathsf{Pred})$	Derived consequence	collects the admissibility, family, geometric, transport, record, and vacuum data used by the closure theorems
Comparison layer	$\mathcal{R}_{\text{cmp}}$, $\mathcal{R}_{\text{SM}}$, $\alpha(0)$, M_Z, $\{m_c, m_b, m_\tau, m_t\}$	Comparison convention	translates closed outputs into scheme-specified observables, with $\mathcal{R}_{\text{SM}}$ only the practical numerical preconditioner

Table 2. Core datum versus theorem, derived-consequence, and comparison-convention structures.

is

$$P_{\text{adm}} = P_{\Sigma,+}\, P_{\text{sing}}\, P_{\Theta}\, P_{\text{gap}},$$

where the factors act on the commuting seam, color, determinant, and transport sectors of the admissible Hilbert-space decomposition. Here $P_{\Sigma,+}$ projects to seam-even sectors, P_{sing} to center-neutral singlets when a hadronic interpretation is intended, P_Θ encodes the declared sheet / determinant selection, and P_{gap} imposes the transport-side positivity or gap condition. The main text defines P_{adm} as the zero-temperature projector of K_{adm}; the appendix records the explicit factorization available on the commuting retained branch.

12 Relative APS and superconnection setup

The geometric completion uses APS-type relative data [1]. In practice this means:

- one works with a declared pair (D, D_{ref}) rather than a single unanchored operator,
- boundary conditions on the seam Σ are part of the definition of the relative index,
- the η-term keeps track of the residual spectral asymmetry,
- the superconnection package is used in the same spirit as the spectral-action tradition [2], but here always with an explicit reference subtraction.

This appendix now serves as the proof motor for the geometric reconstruction statements in the main text: its role is to make the common reference structure explicit enough that the reconstruction theorem and the local spectral-scale branch are not left hanging on a slogan.

13 Source Extraction Map

Source extraction map

Use `../tfpt-42.tex`:

- Sections 1 and 2 only as a shortened claim map.
- Sections 3.1 and 3.2 for primitive core, one-sided boundary datum, doubling, deck

Symbol	First-duty meaning in the theorem chain	Status class
$\mathfrak{T}_\partial^{\min}$	one-sided boundary datum $(\mathcal{A}_+, \mathcal{H}_+, D_+, J, \Gamma, B_\Sigma)$	Boundary datum
$\mathfrak{T}_{\min}^{\mathrm{cl}}$	doubled closed datum $(\mathcal{A}, \mathcal{H}, D, J, \Gamma, \tau_{\mathrm{dbl}}, \iota_C)$ reconstructed from $\mathfrak{T}_\partial^{\min}$	Theorem
$\mathfrak{T}_{\mathrm{prim}}^{\mathrm{kin}}$	primitive kinematic scaffold $(\widetilde{M} \to M, \Sigma, \tau_{\mathrm{dbl}}, \iota_C, D_{\mathrm{ref}}, D_{\mathrm{rel}}, \chi_{\mathrm{seed}})$; when an almost-commutative realization is chosen, S and $\mathcal{H}_{\mathrm{int}}$ refer to that choice and are not additional reconstruction outputs	Theorem
$\mathcal{C}_{\min}$	minimal carrier kernel $E_3 \oplus E_2$ with one-family packet S^+	Theorem
χ_{seed}	primitive seam-even scalar response reconstructed on the primitive layer before geometric reconstruction	Theorem
σ (geometric), χ_{geo}	local Weyl conformal factor and derived local spectral scale $\chi_{\mathrm{geo}} = \Lambda e^{\sigma}$ on the geometric branch	Theorem
σ_{QCD}	QCD string tension $\sigma_{\mathrm{QCD}} = c_3^2 \chi_{\mathrm{QCD}}^2$ on the hadronic transport branch (lexically distinct from the Weyl conformal factor and from the standard-deviation σ in benchmark tables)	Theorem
$\mathrm{APS}_{\mathrm{dom}}^{\mathrm{prov}}$	provisional APS realization of the geometric anchor used in [TFPT cross-reference: `thm:well-posed-primitive-dynamics`]; an analytic starting representative	Boundary datum (analytic choice)
$\mathrm{APS}_{\mathrm{dom}}$	closed-branch APS domain obtained from $\mathrm{APS}_{\mathrm{dom}}^{\mathrm{prov}}$ by the Calderón upgrade [TFPT cross-reference: `thm:apsdom-upgrade`]	Theorem
$\mathrm{APS}_{\mathrm{fam}}$	family-pairing data induced canonically on the doubled admissible branch by the master closure roadmap	Theorem (derived)
P_{prim}	primitive admissibility projector built from ι_C, $\mathrm{APS}_{\mathrm{dom}}$, and D_{rel} alone ([TFPT cross-reference: `def:primitive-admissibility-complex`])	Theorem
$P_{\mathrm{sing}}, P_\Theta$	color-center singlet projector and determinant-sector involution projector available only after carrier and bosonic-index closure	Theorem
$\mathsf{Adm}, K_{\mathrm{adm}}, P_{\mathrm{adm}}$	admissibility predicate, positive constraint operator, and full operative selector $P_{\mathrm{adm}} = P_{\mathrm{prim}} P_{\mathrm{sing}} P_\Theta$ on the admissible branch ([TFPT cross-reference: `thm:padm-upgrade`])	Theorem
F, Ω_{adm}	topological family space and occupancy count after family closure; Ω_{adm} later feeds the closed reading of δ_{top}	Theorem
$[\nabla_F]$	$SU(3)_F$ conjugacy class of the rigid family local system used as the sole family-connection invariant in [TFPT cross-reference: `def:tfpt-rigid-pre-category`]	Theorem
δ_{ph}	unique algebraic transport root on the C_6 hexagon used on the closed branch	Theorem
Q_Θ	determinant-sector involution entering admissibility and the projector P_Θ	Theorem
$\mathcal{C}_\Sigma = \mathcal{C}_\Sigma$	*sheet-CP symmetry operator only*: $\mathcal{C}_\Sigma := CP \circ \tau_{\mathrm{dbl}}$ on the admissible branch	Theorem
T	transport generator on the family / holonomy side	Theorem
$\varpi_{\mathrm{spin}} = \varpi_{\mathrm{spin}}$	spin holonomy weight entering the geometric package	Theorem
$\mathfrak{T}_\star$	canonical rigid object of [TFPT cross-reference: `thm:no-alternatives-tfpt`]; endpoint of the theorem chain	Theorem
$\mathsf{F}_{\mathrm{fals}}(\mathfrak{T}_\star)$	structural falsification map of [TFPT cross-reference: `sec:structural-falsification-map`] (main text, hard-separated from $\mathcal{R}_{\mathrm{cmp}}$)	Theorem (side card)

Table 3. Symbol guide for theorem-level objects only. Comparison-layer and readout symbols are collected separately in Table 4. The previously colliding label $\mathcal{C}_\Sigma$ now denotes only the sheet-CP operator; seam cycle data are written $\mathfrak{C}_\Sigma$ (Table 4).

involution, Calderon projector, and primitive admissibility complex.

- Section 4.1 only for the primitive seam generator, $[u_\Sigma] = 1$, and c_3.
- Sections 8.1–8.4 only for primitive selector and upgrade logic.
- Sections 9.1–9.3 for operational seed, admissible bordisms, and boundary primitive kernel.
- Appendices B and D in shortened technical form.

Editorial guardrail

Every occurrence of carrier rigidity, Standard-Model representations, α, gravity, cosmology, CMB, or E8 should be deleted or converted into a forward reference.

Exported objects

Exports: τ_{dbl}, ι_C, P_{prim}, $[u_\Sigma] = 1$, $c_3 = 1/(8\pi)$, and the boundary primitive kernel $\mathfrak{T}^{\partial}_{\mathrm{ker}}$.

14 Not Used Here

Carrier rigidity, the Standard-Model packet, exact electromagnetic closure, flavor transport, QFT reconstruction, gravity/metrology, cosmology, CMB targets, E8 grammar, horizons, and transient channels are not used as inputs in this paper.

Symbol	First-duty meaning in the appendix layer	Status class
$\mathcal{R}_{\mathrm{cmp}}$	appendix-only comparison map $\mathcal{R}_{\mathrm{cmp}} : \mathfrak{T}_\star \mapsto \mathcal{R}_{\mathrm{cmp}}(\mathfrak{T}_\star)$ of [TFPT cross-reference: `sec:appendix-empirical-readout`]; never enters the theorem chain	Comparison convention
$\mathcal{R}_{\mathrm{SM}}$	practical finite-threshold numerical preconditioner used inside $\mathcal{R}_{\mathrm{cmp}}$	Comparison convention
$\mathcal{I}_{41}$	weighted abelian index $10\,b_1 = \Omega_{\mathrm{occ}}\gamma + N_\Phi$ before the closed 41 specialization (appendix bookkeeping)	Appendix bookkeeping
$\hat{m}_f,\ m_f^{\mathrm{obs}}$	internal source masses and their appendix-level comparison-surface images after matching	Comparison convention
$M_\Sigma,\ g_\Sigma$	seam matching scale read from the first positive seam boundary mode and the gauge coupling evaluated there (used inside $\mathcal{R}_{\mathrm{cmp}}$)	Comparison convention
$\mathfrak{C}_\Sigma = \mathfrak{C}_\Sigma$	seam cycle data used by the matching and holonomy package; lexically distinct from the sheet-CP operator $\mathcal{C}_\Sigma$ in Table 3	Appendix-level derived data
$\mathcal{A}_{\mathrm{rec}}, \mathcal{A}_{\mathrm{obs}}, \mathrm{Pred}$	stable record algebra, observer algebra, and prediction map on the admissible sector (appendix continuation only; not part of the main theorem chain)	Appendix continuation
σ (statistical)	standard-deviation symbol used in benchmark tables for residuals (lexically distinct from the geometric Weyl factor σ and from σ_{QCD})	Appendix bookkeeping

Table 4. Symbol guide for appendix-level comparison and readout objects. These symbols never participate in the main-text theorem chain [TFPT equation: `eq:closed-output-chain`] and are recorded here only because they appear in $\mathcal{R}_{\mathrm{cmp}}$ or in the appendix bookkeeping.

References

[1] M. F. Atiyah, V. K. Patodi, and I. M. Singer, *Spectral asymmetry and Riemannian geometry. I*, Math. Proc. Cambridge Philos. Soc. **77** (1975), 43–69.

[2] A. H. Chamseddine and A. Connes, *The spectral action principle*, Commun. Math. Phys. **186** (1997), 731–750.