A computable, zeta-free truncation family for the Weil measure: measurements on a Hilbert–Pólya candidate

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Abstract

We describe a family of finite, explicitly computable operators — truncations of one glued geometric object built from the Gaussian E_8 lattice — whose spectral data measurably reproduce the Weil measure of prime number theory without ever loading a prime table or a zeta zero. The finite places are the Hecke–commensurability tower of the rank-4 Hermitian unimodular $\mathbb{Z}[i]$-lattice: its primitive degrees *are* the Gaussian prime norms, and a circle-free logarithm generator plus conjugation descent produce the atom comb of the Weil measure at deviation 0.0 [MEASURED]. The archimedean place is a 48-site spin cover lift whose μ_4-fixed heat trace is exactly the density behind $\operatorname{Re}\psi(\frac{1}{4}+i\tau/2)$ — the $\frac{1}{4}$ is derived, not declared — and the two sides glue with *one* normalization: a free three-scalar fit returns $(1,1,1)$ to below 10^{-12}, and among the dimension-8 unimodular lattices $\{\mathbb{Z}^8, E_8\}$ only the Gaussian E_8 glues [MEASURED]. On every truncation the Weil functional is a state (GNS vector state at KMS $\beta=1$); the truncation eigenvalues, frozen by SHA-256 *before* any zero is loaded, hit 100% of the first 377 zeta zeros at tolerance 0.25 with ladder rate -1.61, and the matched-node nearest-neighbour statistic is 0.6178 against the zeros' own 0.6189 [MEASURED]. The finite trace formula of the family is exact Gauss quadrature, and its term dictionary to the classical Weil explicit formula closes block by block at 10^{-13}–10^{-16}. We then decompose what remains: two of the three convergence steps are classical or measured (half-plane convergence with an unconditional Chebyshev majorant, boundary exactly at $s=\frac{1}{2}$; tightness), the node-capture half of the third is proof-near, and the one remaining statement — Weil positivity in the limit — is localized in four machine-verified equivalent languages (Hankel, Levinson, Fejér, Krein–Suzuki), each finitely decidable per window. *No-proof fence:* this note claims no theorem about the Riemann zeta function and no progress on the Riemann Hypothesis at theorem level. Its contributions are the computable zeta-free scaffold, the prediction-freeze methodology, the documented negatives, and the four-language localization of the remaining statement.

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1 Introduction and claim typing

The Hilbert–Pólya heuristic asks for a self-adjoint operator whose spectrum is the sequence of nontrivial zeta zeros. The obstruction is well mapped: Weil's explicit formula [1] identifies the

*Working note of the verification program of Topological Fixed-Point Theory (TFPT), `https://fixpoint-theory.com`. Every number quoted in this note is printed by a machine-checked module of the permanent verification suite (728 scripts, all green at the time of writing): the moonshot arc `v714`, `v716`–`v721` and the keystone round `v727`–`v734` (90 checks), promoted verbatim from exact-arithmetic discovery probes. Zeta zeros enter these modules only as declared comparison targets loaded *after* a printed SHA-256 freeze of all predictions; the construction path is AST-firewalled against prime tables and zero data. No number is quoted that was not printed by an executed run.

zero side with a geometric side (pole, primes, archimedean term), and the Riemann Hypothesis is equivalent to positivity of the geometric functional on autocorrelations; Connes [2] produced a trace-formula realization on the adèle class space in which positivity again carries the full content; Meyer [3] proved the spectral realization unconditionally — *without* positivity, which marks the proved boundary line; Connes–Consani [4] established Weil positivity in the atom-free regime by Sonin-space compression; and Suzuki [5] encoded GRH as positivity of an explicit canonical-system Hamiltonian family, constructed unconditionally exactly on the half-plane parameter range $\omega > 1$.

This note reports a different kind of object inside that map: a *computable, zeta-free truncation family* — finite matrices built from lattice counting and a cover-lift heat trace, with no prime table and no zero data anywhere in the construction path — together with systematic *measurements* of how much of the Hilbert–Pólya picture the family already exhibits at finite size, and machine-checked bookkeeping of exactly what would remain to be proved. All results come from the permanent verification suite of the TFPT repository [21]: the modules `v714`, `v716`–`v721` (the "moonshot arc") and `v727`–`v734` (the "keystone round"), each a standalone script whose checks, bars and verdicts are printed by an executed run.

Every load-bearing claim below is typed:

- [PROVED] — a finite computation certified exactly (integer or rational arithmetic, or a residual at floating-point machine scale), or a one-line classical argument reproduced and machine-instantiated in the module;
- [MEASURED] — a numerical finding with declared bars, error ladders and controls; a measurement, not a theorem;
- [KILLED] — a preregistered hypothesis whose kill criterion fired; documented because negatives constrain the search space;
- [DECLARED] / [CITED] — a normalization chosen and frozen before comparison, resp. classical input used with attribution.

The single most important methodological device is the *prediction freeze*: whenever zeta zeros are used as comparison targets, every prediction of the construction (node positions, peak lists, capture radii, centroids) is serialized and its SHA-256 hash printed *before* the module loads any zero; the zeros live only in isolated `declared_*` functions, and an AST firewall bans prime/zeta identifiers from the construction path.

No-proof fence (stated first, repeated last). Nothing in this note is a theorem about ζ. The spectral agreement of Section 3 is a measurement; the positivity of the truncations is a per-window finite computation, not a limit statement; and the one statement that would close the circle is named in Section 5 and left open.

2 The glued object

2.1 Finite places: the $\mathbb{Z}[i]$-E_8 commensurability tower

Let L be the E_8 root lattice with the root-compatible complex structure J ($J^2 = -1$, $JL = L$), i.e. the rank-4 Hermitian unimodular $\mathbb{Z}[i]$-lattice of [20]. Module `v714` (19/19 checks, verdict MOONSHOT-GO) builds the commensurability tower of L: all $\mathbb{Z}[i]$-submodules of index n, enumerated by Hermite-normal-form cells, graded by the abelian index $n = N(\det)$. Three findings [MEASURED]:

(i) *Primitive degrees are the Gaussian prime norms.* A degree-n layer is primitive iff an index-n submodule admits no proper intermediate module. The machine census over

$n \le 35$ gives the primitive degrees $\{2, 5, 9, 13, 17, 29\}$ — exactly the norms of Gaussian primes, *not* the rational primes: $3, 7, 11, \ldots$ enter only through their squares (inert). Maximality is checked exhaustively at $n = 2, 5, 9, 13$ (15/15, 312/312, 820/820, 4760/4760 maximal) and fails identically at the composite degrees $4, 8, 10$ (0 maximal out of 155, 1395, 4680). No prime projector exists anywhere in the code: the primes come *out* of the tower.

(ii) *The logarithm is generated internally.* The raw layer count is divisorial ($a_5 = 312 = 2(1 + 5 + 25 + 125)$). The groupoid-internal log generator — the derivative of the degree grading, $a_n \log n = \sum_{d|n,\, d\ge 2} \Lambda_A(d)\, a_{n/d}$ — combined with the rank-4 cell normalizer $1 + n + n^2 + n^3$ de-divisorizes the trace: the resulting Λ_{geo} is supported exactly on prime-ideal powers, with on-support relative deviation 1.1×10^{-14} and off-support maximum 4.0×10^{-13} over 20000 slots.

(iii) *Conjugation descent yields the Riemann comb.* Complex conjugation σ acts on the tower; primitive classes split into σ-swapped pairs (split, orbit length $\log N$), ramified ($\log N$) and inert with μ_2 isotropy (half orbit, length $\frac{1}{2} \log N = \log m$). The descended orbit comb — atoms at $u = k\ell$ per orbit base, masses $\mu_n = 2\Lambda_{\text{geo}}(n)/\sqrt{n}$ with the $\sqrt{n}$ half-density [DECLARED] — matches the first 100 atom positions, all 100 masses and 368 window-lag moments of the deployed Weil measure at deviation 0.0.

The honest fence of the slice: the finite counting facts in (i)–(iii) are carrier-generic (E_8-unspecific); the specificity question belongs to the gluing (Section 2.3).

2.2 The archimedean place: cover lift and the derived $\frac{1}{4}$

Module `v716` (18/18, verdict MOONSHOT-STAGE2-GLUED) identifies the archimedean incarnation of the *same* object. The carrier is the 48-site Neveu–Schwarz cover lift: a clock of order 12 with plane $\mathbb{Z}/12 = \mu_4 \times \mu_3$, where μ_4 matches the unit group $\mathbb{Z}[i]^\times = \langle i \rangle$ of the finite tower (the $\mathbb{Z}[i]$ tower *is* the μ_4-fixed part of the $\mathbb{Z}$-commensurability groupoid; character bookkeeping exact, deviation 0.0) and μ_3 carries the deck twists $\nu \in \{\frac{1}{6}, \frac{1}{2}, \frac{5}{6}\}$ [MEASURED]. The $+i$ eigenspace of the half-turn — the modes $m \equiv 1 \bmod 4$ — has heat trace

$$\rho(u) \;=\; \frac{e^{-u/2}}{1 - e^{-2u}},$$

and the machine-verified exact identity (deviation $\le 1.4 \times 10^{-10}$ over the τ-battery)

$$\operatorname{Re}\, \psi\Big(\frac{1}{4} + \frac{i\tau}{2}\Big) = \int_0^\infty \Big[\frac{e^{-2u}}{u} - 2\rho(u)\cos(\tau u)\Big]\, du \tag{1}$$

shows that the deployed archimedean density is the cosine transform of this lift heat trace. Two normalization facts are typed with care: the $\frac{1}{4}$ in (1) is *derived* — it is the μ_4 fixed-sector offset, modes $m \equiv 1 \bmod 4$ giving spectrum $2(k + \frac{1}{4})$ [PROVED] — while $\log \pi$ is the Frullani comparison of the UV reference scale $2 \to 2\pi$ (deviation 0.0), i.e. the self-dual (Tate) normalization of the archimedean place [DECLARED], the exact mirror of the declared $\sqrt{n}$ half-density at the finite places; it is machine-pinned to enter the deployed window only through the $d = 0$ UV cell (at 2.2×10^{-16}). The pole term $2\cosh(u/2)$ is the two boundary exponents $s = 1, 0$. The finite-size operator ladder converges to this continuum object at rate $\sim N^{-2}$: the joint scalar $s^*(N)$ runs $0.99078720 \to 0.99996921$ over $N = 48$–768 with geometric extrapolant $|s^*_\infty - 1| = 2.4 \times 10^{-7}$ [MEASURED]. (That the three digamma channels of the archimedean density are exactly the three deck sectors of this lift was established separately in `v705`, tower traces exact at 3.4×10^{-16}, scalar forced to 1.)

2.3 One normalization; E_8 forced at the gluing

The unified trace functional — the Weil measure candidate — is

$$W \;=\; 2\cosh(u/2)\,du \;-\; \rho(u)\,du \;-\; \sum_n \frac{\mu_n}{2}\big(\delta_{\log n} + \delta_{-\log n}\big), \qquad \mu_n = \frac{2\Lambda_{\mathrm{geo}}(n)}{\sqrt{n}}, \tag{2}$$

built with *no* per-sector scalar: atoms from the tower comb (Gaussian-class sieve to $X = 262144$: 22996 irreducibles, comb slots 23150, missing/extra none), archimedean sector from the lift trace, pole from the boundary exponents. The gluing test of `v716` then measures: the atom-sector scalar $\kappa_{\mathrm{at}} = 1$ exactly on all five windows ($\leq 10^{-12}$); the unified object equals the deployed window vectors at 2.2×10^{-16} over all lags of all five windows; and — the nail — a *free* three-scalar least-squares fit $(\kappa_{\mathrm{at}}, \kappa_{\mathrm{ar}}, \kappa_{\mathrm{p}})$ returns (1.0, 1.0, 1.0) to below 10^{-12}: one object, one normalization [MEASURED]. Must-fail counterfactuals fire as demanded: the μ_2 tower ($\mathbb{Z}$ carrier, all odd modes) leaves residual 0.326, the wrong μ_4 class ($m \equiv 3 \bmod 4$) residual 1.000, the wrong deck-twist channel set misses ρ by 0.474 [KILLED].

Module `v719` (F3) adds the census form of the same statement: the unimodular $\mathbb{Z}$-lattices in dimension 8 are classically exactly $\{\mathbb{Z}^8, E_8\}$ (Mordell [CITED]); both carry a complex structure, the carrier parity fixes the mode-tower period, and in the gluing battery *only* the Gaussian E_8 tower glues ($\kappa = 1$, residual 8.5×10^{-17}), while $\mathbb{Z}^8/\mu_2$ rips at 0.102 and the wrong μ_4 class at 0.314 [MEASURED]. E_8 is not an input of the construction; it becomes forced *at the gluing.*

2.4 Windows and truncations

The measurement frames [DECLARED] are the frame-A window ladder: odd piecewise-linear test functions on the half-integer lattice with bandwidth parameter $D = D(h)$ and depth $M = 2h$, for $h = 142, 203, 344, 488, 606, 878, 1027, 1256, 1433$ (nine windows; a five-window subfamily $h = 184$–1433 with sieve reach $X = 256$ to 262144 is used where reach ladders matter). The window sees W only through its lag vector $p = c_{\mathrm{ar}} + c_{\mathrm{at}} + c_{\mathrm{p}}$ (archimedean integral + atom sums + closed pole lags), and the deployed quadratic form is the odd Toeplitz form $B = \mathrm{odd\_toeplitz}(p)$. The GNS truncation of the multiplication/shift operator is obtained by mapping the trigonometric moments to $[-2, 2]$ via $x = 2\cos\theta$, running the Wheeler (modified-Chebyshev) algorithm to the $K = h$ Jacobi coefficients, and diagonalizing the tridiagonal Jacobi matrix J_K; its eigenvalues are the K Gauss nodes of the window measure, reported in τ-coordinates $\tau_j = \arccos(x_j/2)/D$.

3 Measurements

3.1 The Weil functional is a state on every truncation

Module `v717` (21/21, verdict STATE-ON-TRUNCATIONS) poses convolution positivity exactly on the finite test algebra: with K the unified kernel of (2) and $\tilde{f} = \overline{f(-\,\cdot)}$, the binding identity

$$T(f * \tilde{f}) \;=\; u^{\mathsf{T}}\,\mathrm{Toeplitz}(c)\,u \;=\; 2\,V^{\mathsf{T}} B\,V$$

holds at relative deviation 10^{-12} over an 80-vector battery on all five windows [PROVED] (float-exact identity). Then:

(i) *Constructive state.* The Levinson recursion on the unified moment vector has all reflection coefficients $|k| < 1$ (maximum 0.2376, minimum headroom 0.76): Toeplitz(p) is positive definite at full depth, and T is exhibited as the GNS vector state $\langle e_0, \cdot\, e_0\rangle$ of the truncated translation flow [MEASURED]. The positive-definiteness margins are $\lambda_{\min}(B, G_m) = 2.679 \times 10^{-2}$ down to 1.516×10^{-3} over the five windows.

(ii) *KMS structure.* The degree grading as time evolution (Bost–Connes [6]) gives a Gibbs structure with *one* global half-density $e^{-u/2}$ across all three sectors: the three closed identities (flow partition, boundary characters, orbit counting $\mu_n\sqrt{n} = 2\Lambda_{\text{geo}}(n)$) hold at 2.2×10^{-16}, 5.7×10^{-14}, 1.8×10^{-15}, and evenness of W — the functional equation — becomes detailed balance $M(-u) = e^{-u}M(u)$ at 1.1×10^{-16}: KMS at $\beta = 1$, the Bost–Connes critical temperature, with the $s = \frac{1}{2}$ axis as the symmetric GNS splitting [PROVED] (closed identities). The half-density exponent is measured, not assumed: $-\frac{1}{2}$ fits at residual 8.9×10^{-16}, while $-\frac{1}{4}$ and -1 misfit by 2.577 and 5.153 [MEASURED].

(iii) *Controls.* On the Ihara/Petersen lab (Ramanujan graph), where the trace formula is a theorem, the identical machinery produces an exact state (integer path counts equal the spectral side at 2.7×10^{-9}). On the Epstein control ($x^2 + 5y^2$, discriminant -20, class number 2) the orbit measure itself has negative sites (first at $n = 36$) and the same window machinery suffers Levinson breakdown: T is *not* a state there. The construction measures arithmetic, not an artifact of the pipeline [MEASURED].

The naive regular-representation reading (pole square + diagonal orbit state) does *not* give T: the difference Δ is mixed-sign and, at the generalized minimizers, carries $|\Delta|/\text{margin} = 104\text{–}1563$ — the trace-formula boundary, localized (and resolved classically in Section 3.3).

3.2 Spectrum onto zeros, behind a prediction freeze

Module `v718` (10/10, verdict SPECTRUM-CONVERGES-MEASURED) is the quantitative Hilbert–Pólya measurement. Methodology first: the Gauss-node predictions of all nine windows are serialized and frozen by the printed hash `SHA256 bc78e79f4b39506e...` *before* the declared target section loads 379 zeros ($\gamma \leq 650.669$); the operator itself never sees them. Findings [MEASURED]:

(i) *Hits.* In the matching band ($\tau \in [10, 0.9\pi/D]$, which contains 377 of the loaded zeros), the largest window $h = 1433$ hits 100.0% of the 377 zeros at tolerance 0.25 in τ units, 97.1% at 0.10, 84.1% at 0.05, with mean $|\Delta\tau| = 0.0236$; every window of the ladder scores $\geq 98.4\%$ at tolerance 0.25.

(ii) *Rates.* Over the first 20 zeros the h-ladder mean error falls from 0.0185 ($h = 142$) to 0.0003 ($h = 1433$), log-log slope -1.610; the pure GNS ladder at fixed measure ($K = 179, 358, 716, 1433$) is monotone: $0.4678 \to 0.0153 \to 0.0042 \to 0.0003$. The truncation eigenvalues move *onto* the zeros.

(iii) *The full spectrum, typed.* At $h = 1433$ the non-partnered nodes are exactly the expected GNS quadrature picture: 27 nodes in the pole band $\tau < 14.1$ (trivial-mode mass), 769 of 1271 band nodes are Gauss quadrature nodes in the gaps (node/zero density 3.37), 143 sit at the UV edge. The spectrum is the zero sequence *plus* quadrature filling.

(iv) *Statistics.* As an observation (no gate), the nearest-neighbour ratio statistic of the matched node positions is $\langle r \rangle = 0.6178$ versus 0.6189 for the zeros themselves over 374 values in $[30, 649]$ (GUE 0.5996, Poisson 0.3863 [CITED]).

(v) *Controls.* On the Ihara lab the identical construction rank-saturates and hits the graph spectrum at 2.2×10^{-16} — exact spectral identification at finite K, which is what a proved trace formula looks like. The scramble control (atom positions redrawn uniformly, masses kept, $h = 606$) suffers Levinson breakdown at depth 61: the scrambled measure is not even a state — a structure-borne failure, stronger than any density-chance bound [KILLED].

3.3 The exact finite trace formula and the Weil dictionary

Module `v719` (24/24, verdict SATZ1-LEDGER-EXACT). On every GNS truncation the Gauss quadrature identity

$$\operatorname{tr} f(J_K) \;=\; \sum_j w_j\, f(\tau_j) \;=\; \sum_d a_d\, p_d$$

holds for every band-limited $f(\tau) = \sum a_d \cos(dD\tau)$ of degree $\leq 2K-1$, verified at maximal relative deviation 1.7×10^{-11} over nine windows $\times$ fourteen battery vectors [PROVED]. A ledger example ($h = 1433$, Gauss packet $(0.5, 25)$): spectral side $+2.746337777907$ = pole $+0.004353333627$ + arch $+2.701457620503$ + atoms $+0.040526823777$, deviation 3.2×10^{-14}. The term dictionary to the classical explicit formula [1] closes block by block [PROVED]:

block	classical term of the Weil formula	closure
pole: $\sum_d a_d\, c_{\mathrm{p},d} = \int \tilde a(u)\, 2\cosh(u/2)\, du$	$\hat g(\frac{i}{2}) + \hat g(-\frac{i}{2})$ (residues $s = 0, 1$)	1.7×10^{-13}
atoms: $\sum_d a_d\, c_{\mathrm{at},d} = -\sum_n \frac{\mu_n}{2}\big(\tilde a(\log n) + \tilde a(-\log n)\big)$	prime term $-\sum \frac{\Lambda(n)}{\sqrt{n}}\, g(\log n)$	1.8×10^{-15}
arch: $\sum_{d\geq 1} a_d\, c_{\mathrm{ar},d} = -\int \rho(u)\, \tilde a(u)\, du$	$-\frac{1}{2\pi}\int \hat\varphi\, \frac{\Gamma'}{\Gamma}$-term	3.4×10^{-16}
UV cell: $c_{\mathrm{ar},0}$ = Weil-regularized cell	archimedean regularization	2.2×10^{-16}

Moreover $\omega(\tau) := \operatorname{Re}\psi(\frac{1}{4} + i\tau/2) - \log\pi$ equals $2\theta'_{\mathrm{RS}}$ (Riemann–Siegel theta derivative = smooth zero density = GL(1) scattering phase) at deviation 0.0 [PROVED]; the two-sided (even) reading of the deployed arch block converges on the fine D-ladder to the classical Γ'/Γ term with $\sigma = 1.000161$, $c = -1.0 \times 10^{-4}$ — and, honestly, the *one-sided* coefficient sum has no limit: it drifts by exactly $\log(2)/2$ per D-halving (measured 0.346593/0.346622/0.346609) [MEASURED]. The stage-3 difference term Δ is thereby identified classically: at autocorrelation tests, $D\,\Delta \to [\Gamma'/\Gamma \text{ term}] + [\text{full prime term}]$ — the orbit interference is *not* aliasing; its limit is the classical prime term (rates +2.43/+1.30 on the two packets) [MEASURED].

4 The decomposition: classical, measured, and what remains

The program's ledger splits the missing continuum statement into three convergence steps K1–K3 plus one identification step L1. Modules `v720`, `v721`, `v727`–`v731` adjudicate them.

4.1 K2 is classical on $s > \frac{1}{2}$; the boundary is exactly the critical line

Module `v720` (14/14, verdict K2K3-COLLAPSED-CLASSICAL). For the limit exchange (window evaluations against $e^{-s|u|}\cos(tu)$), tail and discretization errors separate cleanly, with a named unconditional majorant [PROVED] (classical ingredients):

$$|F_h - F_\infty| \;\leq\; 2\, c_{\mathrm{RS}}\, \frac{s + \frac{1}{2}}{s - \frac{1}{2}}\, X_h^{\frac{1}{2} - s} \;+\; C_{\mathrm{disc}}\, D_h^2, \qquad c_{\mathrm{RS}} = 1.03883,$$

the Rosser–Schoenfeld Chebyshev bound $\psi(x) \leq c_{\mathrm{RS}} x$ machine-verified in the sieve range ($x \leq 262144$, maximum 1.03882 at $x = 113$) [CITED]+[PROVED], discretization rates measured $+1.55/+1.94$ ($C_{\mathrm{disc}} \leq 162$). The tail slow-down law is exact: measured tail slopes equal $-(s - \frac{1}{2})$ across the battery $s \in \{1.5, 1.0, 0.75, 0.6, 0.55\}$ (maximum slope deviation 0.000) [MEASURED]. At the boundary itself, $s = \frac{1}{2}$: the $t = 0$ evaluation diverges logarithmically with slope $1.9994 \approx 2$ in $\log X$ (Mertens), while $t \neq 0$ stays oscillatory-bounded ($|S| \in [2.68, 2.83]$ at $t = 25$) — no earlier break. K2 is standard analysis on $s > \frac{1}{2}$; the critical line is the honest boundary, and the fluctuation layer there is exactly where the zeros live (no zeros are loaded in this module).

4.2 K3 collapses to measured tightness

Same module. The truncation states are states of *different* algebras; the common object is a fixed test algebra, and the normalization $T(1) = p_0$ grows like the smooth band density: $p_0 = 0.981 \log(1/D) - 1.306$ (residual 5.8×10^{-3}) — the limit is a σ-finite measure, so the vague topology is forced [MEASURED]. Tightness is then measured: the quadrature weight gauge $\sum_i w_i = p_0$ holds at 2.9×10^{-16}; masses on the compacta $[0, 10)$, $[10, 50)$, $[50, 150)$ are Cauchy along the ladder (last differences 5.8×10^{-7}, 5.4×10^{-4}, 1.7×10^{-3}); the band edge carries only 0.9–5.8% of the smooth ω-density (resolution limit, no anomalous drain); and Gauss τ-window masses identify against the closed zeta-free classical prediction at 1.1×10^{-5}–5.3×10^{-3}, following the $(D\tau_0)^2$ discretization law [MEASURED]. Since weak-$*$ limits of states are positive once no mass escapes, K3 collapses onto K2 + identification: the limit object exists as a positive measure *wherever the ladder is positive definite* — which relocates the entire substance into L1 (Section 5).

4.3 K1: capture is proof-near; the super-resolution is atomicity

Capture [PROVED] (classical chain, machine-instantiated). Module `v721` (14/14, verdict K1-LEMMA-CAPTURE-ONLY) implements Markov–Stieltjes bracketing, the Christoffel variational bound with the correctly *symmetrized* Fejér test polynomial (the unsymmetrized kernel bounds the wrong Christoffel function — measured factor 1.9993, fixed once, documented), and derives the closed capture condition: with cumulative mass C_m and Christoffel envelopes, a node is certified within distance d of t as soon as $L_m(t + d) - R_M(t - d) > 0$. Zero violations occur anywhere (synthetic lab including a merged close pair; every real window). But the certified radii are $d_{\text{pred}} \approx 1.4$–$2.4$ in τ units, while the measured node errors are 3×10^{-4} to 3×10^{-2}: the classical lemma carries the *capture*, not the observed $\sim 900\times$ super-resolution.

Mechanism adjudication [MEASURED]/[KILLED]. Module `v728` (11/13; the two failures are preregistered bars that die honestly, expected outcome K1B-ATOM-PINNING) tests the classical locality/symmetry explanation and kills it (correlations ≈ 0); the competing classical mechanism carries the scale: *atom pinning* of the raw measure — in the synthetic identity lab the node error falls like $K^{-1.84}$ with node-weight ratio $\to 1.03$, and on real data $\log|\text{node} - \gamma|$ correlates at 0.64 with $\log(\varepsilon\,\text{sp}/m)$ (background over mass). Off-line mass, in the toy bridge, shows up as *positivity breakdown* (Levinson death), not as node mislocation: nodes do not mislead, they cease to exist.

The pinning lemma [PROVED]. Module `v730` (12/12) supplies the explicit certified bound: for every x^*,

$$\operatorname{dist}(x^*, \text{nodes}) \;\le\; r_K(x^*) := \frac{\sqrt{g_K}\,|p_K(x^*)|}{\sqrt{K_K(x^*, x^*)}},$$

(residual bound; one-line proof from the three-term recurrence and self-adjointness), sharpened by Kato–Temple (Parlett [CITED]); at an atom of mass m, $K_K(x^*, x^*) \le 1/m$ for all K (Christoffel), giving the $\sqrt{\varepsilon/m}/K$ law (Nevai mass-point attraction [CITED]). Violations: 0 of 54 synthetic configurations, 0 at all 598 symbol peaks and all 535 matched band zeros of the real windows $h = 606, 1433$ — including all 377 frozen targets (freeze `SHA256 52aecc6c7f5a32c8...`); measured actual error follows the *linear* ε/m law, one order below the bound. The sharpest structural finding: on real windows the certificate saturates at the window resolution (Fejér lobe $0.504\,\tau$ at full depth, ladder median 3.27/1.23/0.334/0.533 against it), so *sub-width certification of the measured super-resolution requires raw atomicity beyond every finite window depth* — which is precisely the L1 input, now sharply localized. Supporting lab (`v729` C): one inserted Fejér peak in the real $h = 1433$ measure reproduces the super-resolution scale (spacing/error up to $211\times$, K-rates -2.5 to -4.1) [MEASURED].

The gap layer [KILLED] (strong form). Module `v731` (5/5, verdict GAP-UNIV-KILLED): the sinc-shape deviation of the correlation-normalized Christoffel–Darboux kernel at five gap points plateaus at $0.568 \to 0.389 \to 0.284 \to 0.332$ over $K = 716$–1433 — the preregistered kill bar fires; the strong measured form of bulk (sine-kernel) universality at deployed depth is dead. The Stahl–Totik regularity proxy is clean ($(\prod a_k)^{1/K} = 0.9995$ at $h = 1433$, monotone toward capacity 1) and the fixed-h local Szegő floor is strictly positive on all nine windows, but the uniform-in-h condition decays like $h^{-0.64}$: the h-limit gap layer is governed by the atomic limit — again L1. What survives is a *conditional* fixed-window theorem candidate via Lubinsky, Levin–Lubinsky, Avila–Last–Simon and Totik [7, 8, 9, 10].

5 The wall in four equivalent languages

Module `v727` (11/11, verdict L1-DETERMINED-WALL-TYPED) assembles the identification chain for the vague limit: (a) the limit exists (tightness, Section 4.2) [MEASURED]; (b) the correctly framed moment problem is determinate — in the Gauss-damped Weil-axis frame the damped moments identify against the closed geometric side at maximal relative deviation 1.4×10^{-3} over m_0–m_{16}, and the measured Gauss/Freud growth band $[0.638, 1.117]$ makes the Carleman series divergent: classical, no RH content [CITED]+[MEASURED]; (c) spectral reading equals lag reading machine-exactly per window (4.7×10^{-14}, Gauss exactness) [PROVED]; (d) geometric side equals zero side — the explicit formula, valid unconditionally wherever the zeros lie [CITED].

The apparent paradox — "$T_h \geq 0$ on states, $T_h \to W$ classically, hence $W(g * \tilde{g}) \geq 0$, i.e. the Weil criterion" — resolves honestly: *per-window positive definiteness is a measurement, not a theorem.* The window lags are not a priori the moments of a positive object (the raw symbol dips negative; the scramble control destroys positive definiteness while keeping all masses). The wall, maximally precise:

> **PD persistence.** Infinitely many windows of the ladder are positive definite. This is *sufficient* for the Weil criterion (with (c)-rates and (d)); each instance is a *finite* computation; measured 9/9 with minimum Levinson headroom 0.718. The converse (RH $\Rightarrow$ every finite window PD) is *not* claimed — an honest asymmetry.

Machine-verified on the largest window ($h = 1433$), the same fact in four classical languages, each finitely decidable per window:

language	object	finite witness at $h = 1433$	module
moments	Hankel matrix of the damped moments	min. eigenvalue $1.373 \times 10^{-11} \geq 0$	`v727`
state	Levinson reflection coefficients	headroom $1 - \max \lvert k \rvert = 0.830$	`v727`
symbol	Fejér-smoothed window symbol	minimum $6.475 \times 10^{-3} > 0$	`v727`
Hamiltonian	Krein canonical system, $H_h = h_k h_k^{\mathsf{T}}$	$\min l_k = 0.033$–$0.377 > 0$ (all 9 windows)	`v734`

The fourth language is the Suzuki bridge. Module `v734` (10/10, verdict S1-CANONICAL-SUZUKI-BRIDGED) gauge- normalizes the Jacobi continued fraction into rank-one Krein steps $T_k(z) = I + z\, l_k(-J) h_k h_k^{\mathsf{T}}$: nilpotency residual 3.1×10^{-16}, Weyl identity (canonical chain versus Gauss data) exact at 1.6×10^{-15} [PROVED]; $H_h \geq 0$ is then the statement $l_k > 0$, measured on all windows, and the scramble control breaks the Wheeler/Krein chain at depth 31 — positivity is arithmetic substance, not a formal freebie [MEASURED]. The boundary phase of the canonical system equals the Γ-side $[\omega + \text{pole} - \text{comb}]/\pi$ with scale exactly 1 (deviations 1.9–2.5×10^{-2} at $\tau_0 = 15/25/40$), while the bulk node density is pure Chebyshev equilibrium ($c_1 = 0$ against ω): the arithmetic sits in the boundary phase, not in the bulk — an honest negative sub-finding [MEASURED]. Suzuki [5] encodes GRH as positivity of an explicitly constructed canonical-system family, unconditional exactly for $\omega > 1$ — the same half-plane where our K2 is classical; our truncations are, to our knowledge, the first computable *zeta-free* approximating family with the same Weyl/phase data and a measurable $H \geq 0$ witness per window. "$H \geq 0$ in the limit" is the fourth equivalent reading of PD persistence.

The universal quantifier over the window depth h — and nothing else — is the remaining RH substance of this program.

6 Position in the literature; documented negatives

What the classics own. The explicit formula and the positivity criterion are Weil's [1]. The trace-formula reformulation on adèle classes is Connes' [2]. The proved boundary line is Meyer's [3]: the spectral realization exists *unconditionally*, positivity does not follow — our Section 3 is a finite-size, computable echo of exactly that boundary. Connes–Consani [4] prove Weil positivity in the atom-free support regime ($|u| < \log 2$) by Sonin-space compression; Suzuki [5] owns the canonical-system encoding; Bost–Connes [6] the KMS-$\beta = 1$ phase structure that our detailed-balance measurement reproduces; Lubinsky and Levin–Lubinsky [7, 8] (with [9, 10]) the universality route to clock rigidity that our gap layer conditionally touches and whose strong form our data kill at deployed depth.

The Gate-0 interface [PROVED] (finite census). Module `v733` (13/13, verdict GATE0-WEAK-FUNCTOR): on the finite periodic-orbit Laurent basis for $S = \{\infty, 2, 3, 5\}$, the σ-descended TFPT orbit basis maps into the Laurent orbit subalgebra of Connes' S-local crossed product as a coefficient-one graded $*$-algebra functor candidate — 6859 convolution coefficients at deviation 0, the same scalar $\kappa \equiv 1$ at all three finite places (ramified 2, inert 3, split 5), involution, grading, time action and both traces preserved. Not proved: extension to the full S-local convolution algebra, its topology, adèle coordinates, or the KMS state.

Documented negatives [KILLED] — each preregistered, each now a constraint:

- *Naive reflection positivity is dead on the truncations* (`v729` A, 5/5). The sector anatomy is exact — the pole block is Hankel-PSD (two rank-one squares), the arch block is Hankel-NSD (complete monotonicity of ρ with negative coupling) — and the *total* window fails plain and KMS-twisted reflection positivity at full scale ($\lambda_{\min}/\text{scale} = -1.0$ on $h = 606$ and 1433). The functional-equation symmetry buys exactly the atom-free Connes–Consani regime $|u| < \log 2$ and no more; the deployed positivity lives only on the Toeplitz/Bochner axis.

- *The direct Sonin transfer is dead* (`v732`, all 21 construction checks pass, preregistered kill SONIN-RANK-GROWS fires). Compressing the window forms by the one-cell Sonin complement ($U_0 = \frac{1}{2}\log 2$, $T_0 = \pi/2U_0$, one self-dual phase-space cell) leaves a non-positive remainder whose rank *grows* $19 \to 86$ over the nine-window ladder (largest defect excess 1.3×10^5), and one pole condition is neither stable nor sufficient. Fenced: this kills the *direct discretized transfer*, not the Connes–Consani mechanism itself; tuning U_0, T_0 after the result is forbidden.

- *Strong gap universality is dead at deployed depth* (`v731`, Section 4.3), and the memo locality/symmetry mechanism for the node super-resolution is dead (`v728`) — atom pinning carries the scale instead.

What this work does NOT claim. No statement about the Riemann Hypothesis at theorem level; no continuum self-adjointness, trace-identification or positivity theorem. The contributions are: the computable zeta-free scaffold with emergent primes (Section 2); the measurements with prediction freeze (Section 3); the classical/measured decomposition with the honest boundary at $s = \frac{1}{2}$ (Section 4); the documented negatives above; and the four-language localization of the single remaining statement (Section 5).

7 Verification and reproducibility

7.1 The module ledger

All numbers in this note are printed by the following modules of the permanent suite (each module re-runs its checks on every suite execution; "checks" counts hard gates, not soft reports):

module	checks	verdict	role in this note
`v714_moonshot_hecke_groupoid`	19/19	MOONSHOT-GO	finite places (§2.1)
`v716_moonshot_arch_glue`	18/18	STAGE2-GLUED	arch place, gluing (§2.2–2.3)
`v717_moonshot_state`	21/21	STATE-ON-TRUNCATIONS	GNS state, KMS (§3.1)
`v718_moonshot_spectral`	10/10	SPECTRUM-CONVERGES-MEASURED	zeros, freeze (§3.2)
`v719_moonshot_traceformula`	24/24	SATZ1-LEDGER-EXACT	trace formula (§3.3)
`v720_moonshot_k2k3`	14/14	K2K3-COLLAPSED-CLASSICAL	K2, K3 (§4.1–4.2)
`v721_k1_node_capture`	14/14	K1-LEMMA-CAPTURE-ONLY	capture (§4.3)
`v727_l1_identification`	11/11	L1-DETERMINED-WALL-TYPED	determinacy, wall (§5)
`v728_k1b_superresolution`	11/13*	K1B-ATOM-PINNING	mechanism adjudication (§4.3)
`v729_strat2_rp_universality`	5/5	RP-DEAD/UNIV-EMERGING/PIN-SUPPORTED	kills, pinning lab (§4.3, 6)
`v730_strat2_pinning_lemma`	12/12	pinning lemma, 0 violations	certified bound (§4.3)
`v731_strat2_gap_universality`	5/5	GAP-UNIV-KILLED	gap layer (§4.3)
`v732_strat3_sonin_prolate`	21/21†	SONIN-RANK-GROWS	Sonin kill (§6)
`v733_strat3_gate0_census`	13/13	GATE0-WEAK-FUNCTOR	Connes interface (§6)
`v734_s1_canonical`	10/10	S1-CANONICAL-SUZUKI-BRIDGED	fourth language (§5)

*the two failing bars are the preregistered memo hypotheses whose honest death *is* the verdict; the promoted module gates on exactly this failure set. †construction checks; the preregistered kill criterion fires as the result.

The suite comprises 728 scripts (status ledger: 806 rows), all green under the repository's single entry point (`verification/run_all.py`); the modules above import their measurement frames from the deployed window constructor (`v563`) and re-verify them on every run.

7.2 Lean anchors

The finite algebraic cores that this family rests on are formalized in Lean 4 over Mathlib [18, 19], committed to the repository's Lean project (`lake build` green, kernel-checked, no `sorry`, no `native_decide`): `TfptCarrier/SineGramKeystone.lean` (23 theorems — the sine-Gram keystone decomposition behind the deployed window form) and `TfptCarrier/WatataniIndexFour.lean` (28 theorems — the exact μ_4 index-4 certificate with sharp Pimsner–Popa constant $\frac{1}{4}$), together with the carrier modules `GaussianCodeBridge.lean` (44 theorems) and `QuarticHalf.lean` (21 theorems) formalizing the Gaussian E_8 structure of Section 2.1 — 116 kernel-checked theorems in total. The measurement layer of this note (states, spectra, rates) is deliberately *not* formalized: it is typed as measurement, and its guarantee is reproducibility, not proof.

7.3 Reproducibility

Every module named here is a standalone Python script in `verification/` of the repository [21], runs from a clean checkout with pinned dependencies, prints every check with its bar, and exits nonzero on any failure. The discovery probes from which the modules were promoted verbatim (same numbers) are preserved under `experiments/tfpt-discovery/`. Snapshots of the repository, including the full suite and the Lean project, are archived on Zenodo; the version of record for this note is the August 3, 2026 state (728 scripts).

8 Conclusion

There exists a finite, explicitly computable, zeta-free family of operators — one glued object: Gaussian-E_8 Hecke tower at the finite places, a 48-site cover lift at the archimedean place, one normalization, E_8 forced at the gluing — that is a state on every truncation, satisfies an exact finite trace formula whose term dictionary to the Weil explicit formula closes at machine precision, and whose truncation spectra, frozen before any zero is loaded, measurably converge onto the zeta zeros with quantified rates and GUE-consistent statistics. Around it, the program has now sorted every step of the would-be limit argument into classical (K2, with the boundary exactly at $s = \frac{1}{2}$), measured (K3 tightness; the spectral identification), proof-near (node capture; the pinning lemma with zero violations), killed (naive reflection positivity, direct Sonin transfer, strong gap universality, the locality mechanism), and *open*: one statement — PD persistence of the window ladder, equivalently Hankel positivity of the damped moments, nonnegativity of the Fejér symbol, or $H \geq 0$ for the Krein canonical system — finitely decidable per window, measured true on all nine windows, with the universal quantifier over the depth h as the entire remaining substance.

No-proof fence, repeated. We claim no theorem about the Riemann zeta function and no progress on the Riemann Hypothesis at theorem level. The truncation family, the prediction-freeze measurements, the exact finite trace formula, the documented negatives and the four-language localization are offered as computable scaffolding — a place where the remaining statement can be attacked one finite window at a time, and where any failure would be visible as a single red check.

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[21] TFPT repository (`https://fixpoint-theory.com`): verification suite modules `v714`, `v716`–`v721`, `v727`–`v734` (all under `verification/`); discovery probes under `experiments/tfpt-discovery/` (`moonshot_*`, `k1_*`, `k1b_*`, `l1_*`, `s1_*`, `strat2_*`, `strat3_*`); Lean 4 proof modules under `experiments/lean4-carrier-rigidity/TfptCarrier/` (`SineGramKeystone.lean`, `WatataniIndexFour.lean`, `GaussianCodeBridge.lean`, `QuarticHalf.lean`); archived snapshots on Zenodo (2026).