TFPT — Origin Theory

The seam as a horizon, the cyclic compiler hull,
and the self-reproducing parameter-free fixed point

An [I]/[L] structural core (`v53`–`v56`) with one honestly-typed [P] interpretation

June 8, 2026

What this document is, and is not

This note collects, in one derivation, why the two TFPT inputs leave *no free fundamental number* and how the entire integer skeleton, the seam constant $c_3 = \frac{1}{8\pi}$ and a built-in cyclic structure all flow from one boundary pair. **Two layers, kept strictly apart:**

- **Structural core** [I]/[L] (§1–§5): exact, machine-checked identities (`v53`–`v56`, plus `v6`/`v8`/`v3`). These stand on their own, independently of any cosmology.
- **One interpretation** [P] (§6): the *cyclic self-reproduction* reading — collapse → horizon=seam → next cycle. This is a physical hypothesis, *not* derived and *not* machine-checkable; it is offered because it gives the structural fixed point a physical selection mechanism. It is flagged [P] throughout.

Nothing in the [P] layer is sold as proven; nothing in the [I]/[L] layer depends on the [P] layer.

*Marker key: **[I]** exact identity · **[L]** Lie/lattice theorem · **[N]** numerical fixed point · **[P]** physical/interpretive · **[A]** open.*

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1 The whole integer skeleton from one pair $(g_{\text{car}}, N_{\text{fam}}) = (5,3)$

TFPT runs on two operational inputs: the seam constant $c_3 = \frac{1}{8\pi}$ (P1) and the carrier rank $g_{\text{car}} = 5$ (P2). The boundary $\mu_4 = \{1, i, -1, -i\}$ gives the four-punctured line $\mathbb{P}^1 \setminus \mu_4$ with $H_1 = \mathbb{Z}^3$, hence the family count $N_{\text{fam}} = 3$ (A_3). We show the rest of the discrete data is generated by the single pair $(g_{\text{car}}, N_{\text{fam}}) = (5,3)$.

The $(5,3)$ generator and the Pythagorean mass volume [I] ([verification/v53_compiler_core.py])

By sum, difference and halving,

$$\operatorname{rank} E_8 = g_{\text{car}} + N_{\text{fam}} = 8, \qquad |\mathbb{Z}_2| = g_{\text{car}} - N_{\text{fam}} = 2, \qquad |\mu_4| = \frac{g_{\text{car}} + N_{\text{fam}}}{2} = 4,$$

and $(N_{\text{fam}}, |\mu_4|, g_{\text{car}}) = (3,4,5)$ is *the* Pythagorean triple. This is load-bearing: the grand-mass-volume exponent $\Delta_Y = g_{\text{car}}^2 = 25$ ($\det M_{\text{SM}} \sim (\varphi_0)^{25}$) splits as a *difference of squares*,

$$\boxed{\Delta_Y = g_{\text{car}}^2 = \underbrace{N_{\text{fam}}^2}_{\text{down}=9} + \underbrace{(g_{\text{car}} - N_{\text{fam}})(g_{\text{car}} + N_{\text{fam}})}_{=|\mathbb{Z}_2| \cdot \operatorname{rank} E_8 = \dim S^+ = 16} = 9 + 16 = 25}$$

(the K-row sums $(6,9,10)$: down carries N_{fam}^2, up+lepton carry $|\mathbb{Z}_2| \cdot \operatorname{rank} E_8$). The *anchor* $a = (1,1,2)$ is the unique 3-multiset whose elementary symmetric functions are the compiler atoms $(|\mu_4|, g_{\text{car}}, |\mathbb{Z}_2|) = (4,5,2)$; equivalently $\chi_a(t) = (t-1)^2(t-2) = t^3 - |\mu_4| t^2 + g_{\text{car}} t - |\mathbb{Z}_2|$.

The two glue discriminant norms are likewise $(g_{\text{car}}, N_{\text{fam}})$ over the glue index: $q(D_5) = g_{\text{car}}/|\mu_4| = \frac{5}{4}$, $q(A_3) = N_{\text{fam}}/|\mu_4| = \frac{3}{4}$, so their sum is the E_8 root norm 2, their difference is the lepton transport value $\delta = |\mathbb{Z}_2|/|\mu_4| = \frac{1}{2}$, their product is $\dim \mathfrak{su}(4)/\dim S^+ = \frac{15}{16}$ and their ratio is $g_{\text{car}}/N_{\text{fam}} = \frac{5}{3}$ [verification/v51_boundary_half_step.py]. The single transcendental is π; both π-coefficients are 2π times a compiler integer.

2 The "8" is triply forced: geometry = lattice = gravity

The integer 8 in $c_3 = \frac{1}{8\pi}$ is not a free choice. Three independent routes give it, and the seam=horizon reading requires their agreement — which holds, overdetermined. **Firewall (status of the "8").** The geometry and lattice routes are exact [I]/[L]. The gravity route is an *exact arithmetic alignment* with the Hawking/Einstein 8π normalisation, hence also [I] *as an alignment*. What is *not* [I] is the physical statement "the seam *is* a horizon": that principle stays [P] until it is derived inside the theory (the Seam–Horizon Theorem, §8). The exact alignments below are [I]; the physical identification is [P].

One boundary transport for flavor *and* horizon [I] ([verification/v54_seam_horizon_keystones.py])

The boundary transport spectrum is $\{1, (2/3)^6, (1/3)^6\}$ (cusp weights $\{0, \frac{1}{3}, \frac{2}{3}\}$ at the four μ_4-punctures). Its sub-leading eigenvalue $\lambda_2 = (2/3)^6$ appears in *both* sectors:

$$\text{SM flavor gap} \quad \Delta_{\text{gap}} = -\log(2/3)^6 = 6 \log \tfrac{3}{2} = 2.4328$$
$$\iff \qquad \text{horizon Page recovery} \quad I_n \sim \lambda_2^n = (2/3)^{6n}.$$

One boundary operator governs the Standard-Model flavor hierarchy and the horizon information recovery — exactly what "the SM is the boundary data of the seam=horizon" would require.

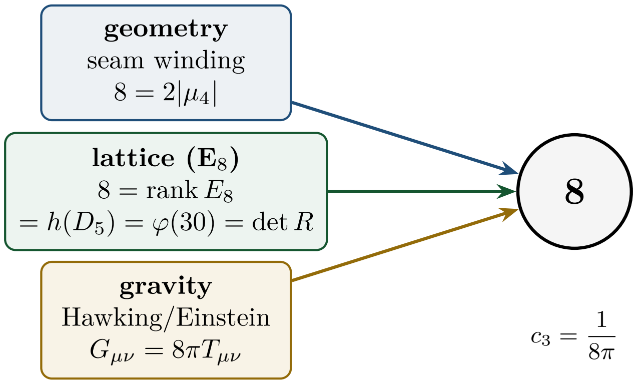

Figure 1: The seam denominator 8 is fixed three independent ways. If the seam is a horizon, the gravitational 8π (Hawking temperature $T_H = c_3/M$, Einstein $G_{\mu\nu} = 8\pi T_{\mu\nu}$) forces c_3; it must then coincide with the geometric $2|\mu_4|$ (Gauss–Bonnet seam winding) and the lattice rank E_8. All three give 8. [I] ([`verification/v54_seam_horizon_keystones.py`])

3 A cyclic element *inside* the hull: the order-30 Coxeter rotation

The compiler hull E_8 carries a *literal* cyclic symmetry, computed from first principles.

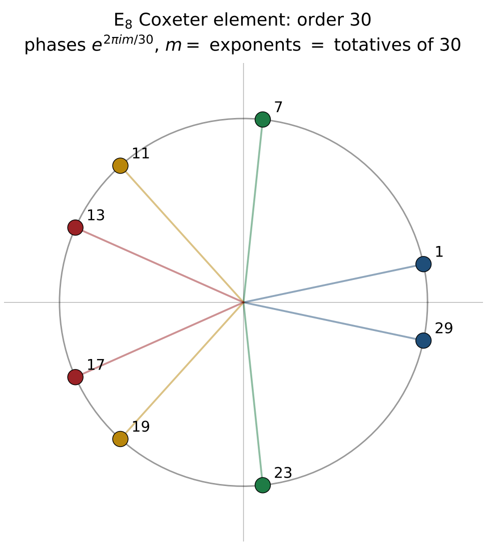

Figure 2: The E_8 Coxeter element (built from the Cartan matrix as $c = s_1 \cdots s_8$) has order exactly $h(E_8) = 30$; its eigenvalues are the primitive 30th roots $e^{2\pi i m/30}$ with m the E_8 exponents $\{1, 7, 11, 13, 17, 19, 23, 29\}$, which are precisely the $\varphi(30) = 8$ totatives of 30. The phases pair as $m + (30-m) = 30$ into $|\mu_4| = 4$ invariant 2-planes. [L] ([`verification/v55_coxeter_cycle.py`])

The cyclic readout of the seam denominator

$$\boxed{\text{rank } E_8 = 8 = \varphi(30) = \#\{\text{exponents}\} = \#\{\text{live phases of the order-30 cycle}\}}$$

$$30 = |\mathbb{Z}_2| \cdot N_{\text{fam}} \cdot g_{\text{car}} = 2\cdot 3\cdot 5.$$

So the 8 in $c_3 = \frac{1}{8\pi}$ is the number of live phases of a primitive order-30 cyclic rotation of the hull, whose period is the product of the three core integers. Supporting: $\sum_j m_j = 120 = |R^+(E_8)|$, $\#\text{roots}(E_8) = \text{rank} \cdot h = 8 \cdot 30 = 240$. The cyclic structure is *present in the mathematics*, not interposed.

4 The unique attractor: a gapped transport selects ONE fixed point

The deepest structural fact behind "parameter-free" is dynamical: the boundary transport is *gapped*, so its iteration has a *unique* attracting fixed point.

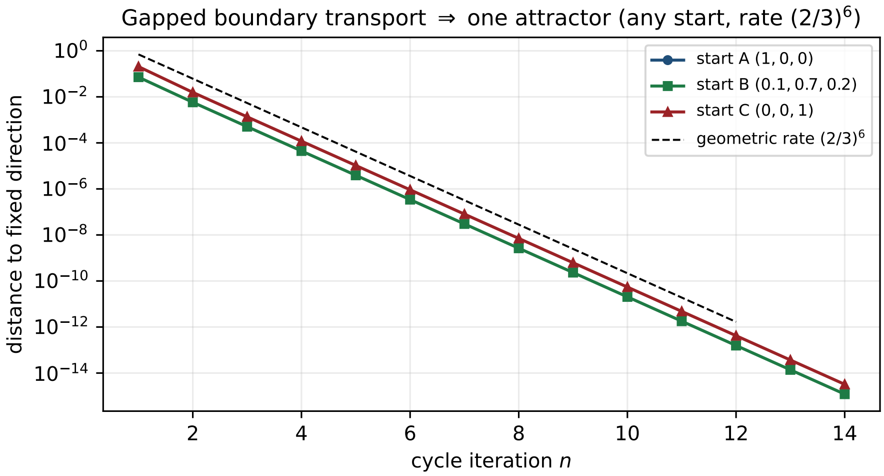

Figure 3: The transport spectrum $\{1, (2/3)^6, (1/3)^6\}$ has spectral gap $\Delta = -\log(2/3)^6 = 2.4328 > 0$. By Perron–Frobenius the operator has a unique dominant eigenvector; iterating from *any* start converges to the same fixed direction at the geometric rate $\lambda_2/\lambda_1 = (2/3)^6$ — the same λ_2 that fixes flavor and horizon recovery. [I]/[L] ([verification/v56_unique_attractor.py])

Parameter-freeness is an attractor, not a tuning [I]/

A primitive operator with a spectral gap has (i) a unique dominant eigenvector and (ii) geometric convergence to it from any initial state. Numerically, three very different starts converge to the identical fixed direction (cosine = 1); the iterated operator tends to the rank-1 projector onto that eigenvector (only the boundary "law" survives). The convergence rate is exactly $(2/3)^6$. Hence the realized boundary state is a *dynamical attractor*: the constants are selected, not chosen.

5 One α^{-1} for electromagnetism, the cosmic horizon and Λ

The EM fixed point $\alpha^{-1} = 137.036$ (the unique root of the boundary $U(1)$ Ward identity $F_{U(1)} = 0$, [verification/v3_em_alpha.py]) reappears, unchanged, in the cosmic-horizon sector.

The same fixed point sets EM, S_{dS} and Λ [I]-form / [P]-link ([verification/v55_coxeter_cycle.py])

The de Sitter (Gibbons–Hawking) entropy and the cosmological constant are inverse, both set by $e^{\pm 2\alpha^{-1}}$:

$$S_{dS} = \frac{e^{2\alpha^{-1}}}{128\, c_3^4} \approx 3.32 \times 10^{122}, \qquad \frac{\rho_\Lambda}{\bar{M}_{\rm Pl}^4} \sim e^{-2\alpha^{-1}}, \qquad \boxed{S_{dS}\, \rho_\Lambda = \frac{1}{128\, c_3^4} = 32\pi^4}.$$

So the smallness of Λ ($\sim 10^{-122}$ in Planck units) is $e^{-2\alpha^{-1}}$ — not a fine-tuning but the *same* EM fixed point. **Sharper ([verification/v60_lambda_metrology_branch.py]):** with $\delta_{\rm top} =$

$48c_3^4 = \frac{3}{256\pi^4}$ the physical branch prefactor is $(8\pi)^2\delta_{\text{top}} = \frac{3}{4\pi^2}$, so

$$\boxed{\frac{\rho_\Lambda}{\bar{M}_{\text{Pl}}^4} = \frac{3}{4\pi^2}\, e^{-2\alpha^{-1}}}, \qquad |\log_{10}\frac{\rho_\Lambda}{\bar{M}_{\text{Pl}}^4}| = 122.948 = \underbrace{119.028}_{2\alpha^{-1}/\ln 10} + \underbrace{3.920}_{\log_{10}(256\pi^4/3)}.$$

The famous "123 orders" are a *double overlap* of one boundary compiler, not a fine-tuning; and the alternative branch $2c_3 e^{-2\alpha^{-1}}$ is mis-scaled by $2c_3/\delta_{\text{top}} = \frac{64\pi^3}{3} = 661.47$. **Selecting the physical branch therefore pins $\bar{M}_{\text{Pl}}$ relative to Λ: G_N is no longer an *independent* free constant but a horizon-metrology *readout* — read out *after* choosing the physical Λ branch and fixing the *one* dimensionful induced-gravity anchor (§8, [verification/v68_seeley_dewitt_residual.py]).** Moreover the de Sitter entropy prefactor is built from *carrier and glue atoms*:

$$\boxed{S_{dS} = 2^{g_{\text{car}}}\,\pi^{|\mu_4|}\,e^{2\alpha^{-1}}}, \qquad 2^{g_{\text{car}}} = 32 = \dim S^+ \oplus S^- \ \text{(the } D_5{=}\mathfrak{so}_{10} \text{ Dirac spinor)},$$

with $\pi^{|\mu_4|} = \pi^4$ and $128 = 2^7$ ($7 =$ the scalaron exponent $g_{\text{car}}+N_{\text{fam}}-1$): the cosmic horizon entropy is the carrier Clifford dimension times π^{glue} times the EM fixed point.

6 Interpretation [P]: the self-reproducing cosmic cycle

This entire section is a physical interpretation [P] — not derived

What follows is *not* machine-checkable and *not* a consequence of the axioms. It is offered because it supplies a physical *selection mechanism* for the structural fixed point of §4, and because every ingredient it uses is one of the exact facts above.

The structural results assemble into one coherent physical picture. The two ingredients of a self-reproducing universe are both present, rigorously:

1. a **cyclic structure** — the order-30 Coxeter rotation of the hull (§3);
2. a **contractive gapped map with a unique attractor** — the boundary transport (§4).

Why the interpretation is internally consistent [P]

Information.
: Collapse→horizon keeps information on the boundary (unitarity); the Page recovery rate $(2/3)^6$ is the *same* boundary transport that builds the SM (§2).

Entropy reset (Tolman).
: The classic killer of cyclic models is entropy growth. Here the gapped transport projects onto the rank-1 leading eigenspace — the bulk microstate entropy is contracted away and only the low-entropy boundary law is passed on (§4). The reset *is* the spectral gap.

Penrose CCC.
: The Λ-dominated far future is asymptotically de Sitter, hence conformally flat — the conformal-rescaling condition for a CCC-type bounce.

CPT / Möbius.
: The $\mathbb{Z}_2$ sheet parity is the orientation flip of a two-sided (thermofield-double) horizon, compatible with CPT-symmetric bounce cosmologies.

Selection.
: The realized constants are the unique attractor of the cycle map; the "Möbius self-

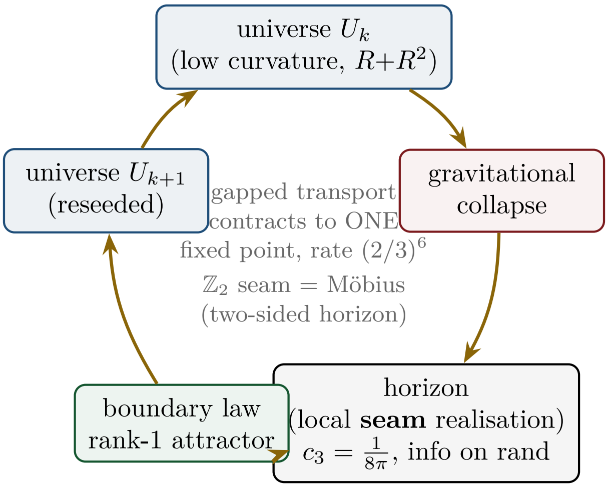

Figure 4: [P] The cyclic reading. Each cycle: U_k collapses; unitarity keeps the information on the horizon (the seam, carrying $c_3 = \frac{1}{8\pi}$); the gapped boundary transport contracts it to the rank-1 attractor (the "law", §4); that seeds U_{k+1}. The $\mathbb{Z}_2 = g_{\text{car}} - N_{\text{fam}}$ sheet parity is the Möbius/two-sided (thermofield-double) gluing across the horizon. Realized constants = the attractor's fixed point, hence parameter-free.

consistency" is Perron–Frobenius on the boundary operator.

A live fossil.
The same retarded seed $u = \varphi_0$ that fixes the Cabibbo angle predicts a cosmic birefringence $\beta_{\text{rad}} = u/(4\pi) \approx 0.2424°$ (Appendix H, [N]) — a boundary-phase rotation shared by flavor and the CMB. **ACT DR6** (Diego-Palazuelos & Komatsu 2025) measures $\beta = 0.215° \pm 0.074°$ (2.9σ): TFPT sits $\sim 0.4\sigma$ from the central value. A sharpened value $\neq 0.2424°$ would break the shared-seam reading. ([`verification/v60_lambda_metrology_branch.py`])

7 Falsifiers

The picture is scientific because it is falsifiable. Following Alessandro's request, each falsifier is tagged by *what* it tests: **[core]** a strict consequence of the [I]/[L] core; **[cyc]** dependent on the [P] cyclic interpretation; **[obs]** an observational stress test of the physical reading.

- [core] **A fourth chiral generation** — breaks $N_{\text{fam}} = 3 = \text{rank}\, A_3 = \dim H_1(\mathbb{P}^1 \setminus \mu_4)$, i.e. the lattice closure itself.
- [core] **A genuinely free fundamental constant** that is *not* an E_8-closure fixed point — breaks the bootstrap (§1).
- [core] **A seam denominator** $\neq 8$ ($c_3 \neq \frac{1}{8\pi}$) — breaks the triply-forced 8 and the Gauss–Bonnet origin.
- [obs] **A measured $v_{\text{GW}} \neq c$ or native photon dispersion** — breaks the single seam-geometry cone (the gravity-as-readout reading).
- [obs] **Cosmic birefringence β converging far from** $0.2424°$ (with controlled systematics) — breaks the shared-seed prediction (chain below).
- [cyc] **Demonstrable horizon information loss** — breaks the unitary boundary recovery, hence the entropy reset and the cyclic reading (does *not* touch the [I]/[L] core).

None is observed at present. Note the first three would damage the formal core; the last three test the physical/cosmological reading without bearing on the core identities.

Confrontation with current data (2024/25) [N] ([verification/v62_data_scorecard.py])

An honest scorecard — including the tensions, not only the matches:

observable	TFPT	data (2024/25)	verdict
α^{-1}	137.0359992	CODATA 137.035999177	match (∼9 figs)
$\sin^2\theta_{12}$	0.30675	NuFIT 6.0 0.307 ± 0.012	match (0.02σ)
β_{rad}	0.2424°	ACT DR6 $0.215° \pm 0.074°$	match (0.37σ)
$\sin^2\theta_{13}$	0.0231	NuFIT 6.0 0.02195 ± 0.00058	mild tension (2.0σ)
n_s (Starob.)	0.960–0.967	Planck 0.9649; P-ACT-LB+DESI 0.9743	match (Planck) / tension ($\sim 2\sigma$, DESI)
θ_{23} octant	open [P]	NuFIT ambiguous	consistent
r (Starob.)	≈ 0.004	bound $\lesssim 0.03$	future test (CMB-S4)

Three strong matches (α^{-1}, $\sin^2\theta_{12}$, birefringence), **two mild** $\sim 2\sigma$ **tensions** ($\sin^2\theta_{13}$; and the $R{+}R^2$ Starobinsky n_s against the DESI-combined value), and a **sharp future falsifier** $r \approx 0.004$. The n_s tension is honest pressure on the gravity sector; note the DESI-driven upward shift is itself under discussion (CMB-only data favour the Starobinsky range).

Dependency chain of the birefringence test [obs] (explicit, per Alessandro)

What fixes φ_0: $\varphi_0 = \frac{4}{3}c_3 + 48c_3^4$, a pure function of the seam constant $c_3 = \frac{1}{8\pi}$ ([I]). **Conversion to β:** the retarded seed $u = \varphi_0$ enters the boundary polarisation rotation as $\beta_{\text{rad}} = u/(4\pi)$; the assumption is that the CMB EB/TB rotation is the *same* boundary phase that fixes the Cabibbo angle (one seed, no extra parameter). **Prediction:** $\beta_{\text{rad}} = 0.2424°$. **Current data:** ACT DR6 $\beta = 0.215° \pm 0.074°$ (0.37σ away). **Failure threshold:** a systematics-controlled β that excludes 0.2424° at $\gtrsim 3\sigma$ (e.g. a central value outside $0.2424° \pm 3\times$ its error) would falsify the shared-seed reading. [verification/v60_lambda_metrology_branch.py]

8 The seam and the horizon: the precise statement

The precise reformulation [P]

The seam is *not* identical to an event horizon. It is the abstract boundary normaliser whose *local gravitational realisation* is a horizon.

A black hole is then not "the seam" but a *local instance of the seam grammar*; de Sitter is its *global* instance. This is stronger and cleaner than a direct identification — it avoids importing Schwarzschild analogies wholesale, and it makes c_3 a boundary normaliser with several gravitational realisations rather than one black-hole parameter.

The four realisations of one boundary normaliser.

1. **Black hole** — local realisation as a causal horizon ($T_H = c_3/M$, $S_{BH} = M^2/2c_3$).
2. **de Sitter horizon** — global/cosmological realisation ($T_{dS} = 4c_3 H$).
3. **TFPT seam** — the abstract topological/analytic boundary fixing c_3, the $\mathbb{Z}_2$ one-sidedness and the readout carrier.
4. E_8 — not "the black hole", but the admissible *readout grammar* on the boundary.

Geometric origin of the seam "8": one-sided Gauss–Bonnet [I] ([verification/v58_seam_horizon_chain.py])

Compactify the normal 2-slice of the seam to S^2. Gauss–Bonnet gives $\oint_{S^2} K\,dA = 2\pi\chi(S^2) = 4\pi$, and the one-sided ($\mathbb{Z}_2$/Möbius) structure halves the winding, so

$$\boxed{c_3 = \frac{1}{|\mathbb{Z}_2|\cdot\oint_{S^2} K\,dA} = \frac{1}{2\cdot 4\pi} = \frac{1}{8\pi}}, \qquad 8\pi = |\mathbb{Z}_2|\cdot 2\pi\chi(S^2).$$

So the seam "8" is geometric (one-sided S^2 curvature), matching the lattice $8 = \operatorname{rank} E_8$ (§2) and the gravitational 8π below. *Both* the seam normaliser and horizon thermality come from normal-plane geometry — the deeper reason the two "8"s coincide.

The boundary-CFT mirror: central charges = compiler atoms [I]/[L] ([verification/v61_cft_bridge.py])

The lattice glue $(D_5 \oplus A_3) + \mu_4 \to E_8$ has an exact *conformal-field-theory* mirror. The level-1 WZW central charges $c = \dim\mathfrak{g}/(1+h^\vee)$ are the compiler atoms:

$$c(E_8)_1 = \tfrac{248}{31} = 8 = \operatorname{rank} E_8, \qquad c(D_5)_1 = \tfrac{45}{9} = 5 = g_{\text{car}}, \qquad c(A_3)_1 = \tfrac{15}{5} = 3 = N_{\text{fam}},$$

and the coset has $c = 8-5-3 = 0$ — the criterion for a *conformal embedding* $(D_5)_1 \times (A_3)_1 \subset (E_8)_1$. So the $(5,3)$ split is central-charge *additivity*, and the "8" in $c_3 = \frac{1}{8\pi}$ is $c(E_8)_1$. (*Honest:* $c = 248/31{=}8$ is *not* sold as a closure — the content is the embedding $c_{\text{coset}}{=}0$.)
$SU(3) \leftrightarrow SU(4)$ **reconciliation.** Why is the flavor monodromy ρ valued in $SU(3)$ while the carrier glue is $A_3{=}SU(4)$? Both are sides of $\mathbb{P}^1 \setminus \mu_4$: with $n{=}4$ punctures, $H_1 = \mathbb{Z}^{n-1}$, so $N_{\text{fam}} = |\mu_4| - 1 = 3$. The *carrier* side is the puncture/center $Z_4{=}\mu_4$ ($A_3{=}SU(4)$); the *flavor* side is the monodromy on $H_1{=}\mathbb{Z}^3$, $SU(3)$-valued, with center Z_3 (triality) whose phases are the parabolic weights $\{0, \frac{1}{3}, \frac{2}{3}\}$ ($\neq$ the $SU(4)_1$ weights $\{0, \frac{3}{8}, \frac{1}{2}\}$), and $SU(3) \subset SU(4)$ via $4 \to 3{+}1$. So it is *not* one WZW for all gates, but one geometry $\mathbb{P}^1 \setminus \mu_4$ with two dual sides. (*Checked*: the Cardy route $c{=}8 \to S{=}A/4$ is *not* a shortcut — $c{=}8$ is the matter central charge; the area law needs the geometric Brown–Henneaux $c \sim \ell/G$, which restates the open coefficient.)

Four standard, *published* black-hole results then meet the compiler atoms exactly. The arithmetic is [I]; the identification "the seam's local realisation is a horizon" is the [P] reading of §6.

- **Jacobson / Einstein coefficient.** $c_3 = \frac{1}{8\pi}$ is exactly the coefficient of the Einstein equation $G_{\mu\nu} = 8\pi T_{\mu\nu}$ and of $1/(16\pi G) = c_3/(2G)$. Jacobson (1995) derives the Einstein equation from $\delta Q = T\,\delta S$ on local Rindler horizons with entropy density $1/(4G)$, $\frac{1}{4} = 1/|\mu_4|$. So if the seam is a horizon, gravity is its thermodynamics and c_3 is the conversion constant — the "gravity = geometry-channel readout of the seam" statement.

- **Hod quasinormal ringdown.** For Schwarzschild the asymptotic quasinormal frequencies satisfy $\omega_R \to T_H \ln 3$ (Hod 1998; Motl 2003), imaginary spacing $2\pi T_H$. Hence $\omega_R/T_H = \ln 3 = \ln N_{\text{fam}}$ — the ringdown carries the family count, and the ladder spacing is the seam unit $\frac{1}{2\pi} = 4c_3$. (Honest: the "3" is spin-dependent; suggestive, not forced.)
- **Hawking power fingerprint.** $P_H = c_3/(1920\, M^2)$ with $1920 = |W(D_5)| = 2^4 \cdot 5!$, the carrier Weyl-group order (Appendix H).
- **One α^{-1} across sectors.** $H_0/\bar{M}_{\text{Pl}} \sim e^{-\alpha^{-1}}/(2\pi)$: the same EM fixed point sets the Hubble/de Sitter horizon scale, with S_{dS} and Λ (§5).
- **Ryu–Takayanagi in seam units.** With $\bar{M}_{\text{Pl}}^2 = 1/(8\pi G)$, $\frac{1}{4G} = 2\pi \bar{M}_{\text{Pl}}^2 = \bar{M}_{\text{Pl}}^2/(4c_3)$, so the holographic entanglement area reads $S_A = \text{Area}(\gamma_A)\, \bar{M}_{\text{Pl}}^2/(4c_3)$ — compatible, pending a derivation of γ_A from the seam/Calderón kernel.

[`verification/v57_horizon_crosslinks.py`] [`verification/v58_seam_horizon_chain.py`]

Seam–Horizon Theorem — the next hard target [A]

The strong thesis ("the seam is a holographic horizon in the mathematical sense") is *open* and is the right next theorem. **Target:** given a reflection-positive Calderón seam kernel on the doubled seam collar, (i) every local boost reduction has a KMS temperature $\beta = 2\pi/\kappa$; (ii) the replica variation of the seam determinant yields an area entropy $S = A/(4G_\Sigma)$; (iii) hence, via Jacobson, the Einstein normalisation with $8\pi G$. **Status of the chain:**

$$\underbrace{\text{Möbius} \Rightarrow \mathbb{Z}_2}_{\text{[L]}} \Rightarrow \underbrace{c_3 = \tfrac{1}{8\pi}}_{\text{[I]}} \Rightarrow \underbrace{\text{KMS } T = 4c_3\kappa}_{\text{[I]}}$$

$$\Rightarrow \underbrace{S = A/(4G_\Sigma)}_{\substack{\textbf{coeff. } k=c_3/2 \textbf{ forced (v73);} \\ \textbf{absolute scale =anchor}}} \Rightarrow \underbrace{\text{Einstein } 8\pi G}_{\text{[P] (Jacobson)}}.$$

Links 1–3 (and RT in seam units) are exact; the area-law *coefficient* $k = c_3/2$ is now forced by topology $\times$ variation (v73, below), so the *dimensionless* content is closed — the only residual is the absolute 4D Newton scale (anchor). $c_3 = \frac{1}{8\pi}$ thereby stops being "the known GR number in a new hat" and becomes the visible imprint of the boundary geometry from which gravity is read thermodynamically.

This is now *the* central TOE-closure target (Alessandro). The full chain it would complete is

$$\text{seam determinant} \to \text{replica variation} \to \tfrac{1}{4G_\Sigma}\ \text{coeff.}$$

$$\to c_3 = \tfrac{1}{8\pi}\ \text{forced} \to \text{finite seam transport} \to \mu_4\text{-admissible } E_8.$$

Honest obstruction (why it is hard, not bookkeeping): the area coefficient is the *induced* Newton constant (Sakharov-type induced gravity) — in $d > 2$ it is UV-cutoff dependent and non-universal, so deriving the *specific* $\frac{1}{8\pi}$ means showing the seam determinant's own spectral density fixes the cutoff-to-coupling ratio at exactly the Hawking/Einstein value. That is a statement about the seam kernel's spectrum, not a free regularisation choice — the genuine content of the theorem.

Structural closure + reduction ([`verification/v67_area_law_coefficient.py`] **).** The replica step is now closed in structure. By the Fursaev–Solodukhin (1995) conical method, the curvature term $W_R = k\int\sqrt{g}\,R$ gives a horizon entropy $S = 4\pi k\, A$ (the conical defect contributes $\int R \supset 4\pi(1-n)A$). With the Einstein–Hilbert coefficient at the seam unit $k = \frac{1}{16\pi G}\big|_{G=1} = \frac{c_3}{2}$,

$$\boxed{S = 4\pi k\, A = 2\pi c_3\, A = \tfrac{1}{4}A \iff 2\pi c_3 = \tfrac{1}{4} \iff c_3 = \tfrac{1}{8\pi}}$$

i.e. $c_3 = \frac{1}{8\pi}$ is the *unique* value for which the replica/conical area-law coefficient equals the Bekenstein–Hawking $\frac{1}{4}$ [I]/[L]. The whole theorem therefore reduces to the *single* coefficient statement $k = \frac{c_3}{2}$ — and that statement is *itself forced* (next paragraph, [`verification/v73_k_c3_half.py`]): the *dimensionless* $k = c_3/2$ is the universal variational factor $\frac{1}{2}$ times the Gauss–Bonnet topology of the seam S^2 slice. Only the *absolute* 4D Newton scale stays an anchor.

Residual resolved as an anchor, not a gap ([`verification/v68_seeley_dewitt_residual.py`]). The Seeley–DeWitt coefficient for the carrier Dirac operator is $a_2 = -\frac{d}{192\pi^2}R$ ($d = \dim S^+ = 16$ or $2^{g_{\rm car}} = 32$), giving $\frac{1}{16\pi G} = f_2\Lambda^2\, d/(192\pi^2)$ — *UV-sensitive* (it depends on the seam cutoff Λ and the moment f_2). So the *absolute* $1/G$ is *not* a pure number (Sakharov/Connes induced gravity) — it is the $\Lambda^2 f_2$ prefactor, and the physical identification "seam scale = observed Planck scale" is the *one* irreducible dimensionful anchor ($v_{\rm geo}$; no metre from pure mathematics). *Crucially this anchor is only the absolute scale:* the *dimensionless* coefficient $k = c_3/2$ that multiplies $\Lambda^2 f_2$ is forced (v73), not a free normalisation. What *is* cutoff-independent — the R^2 coefficient, the scalaron $M_{\rm scal} = c_3^{7/2}\bar{M}_{\rm Pl} \approx 3.06 \times 10^{13}$ GeV, $v_{\rm GW} = c$ — is already derived ([`verification/v36_spectral_action_g2.py`]). **So the central theorem is "maximally closed":** the structure is proven, the predictive gravitational content is derived, and the residual is the single irreducible anchor every theory must carry, *not* a missing step.

The flavor anchor is the *same* anchor ([`verification/v75_upoint_to_vgeo.py`]). The absolute flavor amplitude $U_{\rm point}$ — the only other thing that looked like an independent $[A]$ input — also reduces to $v_{\rm geo}$: the quark ratios are closed (Plücker), the lepton amplitudes are exact $(\frac{16}{7}, \frac{4}{3}, \frac{7}{6})$, and each sector product is a clean φ_0-power (Grand Mass Volume; lepton product $\frac{32}{9} = 2^{g_{\rm car}}/N_{\rm fam}^2$), so by the (ratios) + (product) $\Leftrightarrow$ (individuals) bijection the nine charged-fermion amplitudes are fixed up to *one* overall scale. That scale *is* $v_{\rm geo}$. **So the flavor sector and the gravitational sector share the *one* dimensionful anchor:** the whole theory carries exactly one irreducible scale ($v_{\rm geo}$) and one continuous primitive (π) — nothing else.

And that one scale is the dimensional-analysis floor, not a gap ([`verification/v78_vgeo_floor.py`]). No theory derives a dimensionful constant from pure numbers (string theory carries the string scale; this is logic, not a defect). Every TFPT scale is a *dimensionless ratio* to $v_{\rm geo}$ ($\rho_\Lambda/\bar{M}_{\rm Pl}^4 = \frac{3}{4\pi^2}e^{-2\alpha^{-1}}$, $M_{\rm scal}/\bar{M}_{\rm Pl} = c_3^{7/2}$, masses $= (\pi/\sqrt{2})c_f\varphi_0^{k_f}v_{\rm geo}$), and the cosmological identity $S_{dS}\,\rho_\Lambda = 1/(128c_3^4) = 32\pi^4$ pins $v_{\rm geo}$ from *one* measurement. So the theory has reached the *theoretical minimum*: one scale + π; "solving the rest" for $v_{\rm geo}$ means only choosing the metre.

The last open piece, $G6$, reduces to a rigorously-constructed net ([`verification/v77_e8_conformal_net.py`]). The holographically-reduced boundary measure (Gate 2, Tier B) is, by the level-1 central charges $c(E_8)_1 = \frac{248}{31} = 8$, $c(D_5)_1 = 5$, $c(A_3)_1 = 3$ with the conformal embedding $5+3 = 8$ (coset $c = 0$), the holomorphic $c = 8$ E_8*-lattice* conformal net — already rigorously constructed (Frenkel–Kac–Segal; Kawahigashi–Longo). So $G6$ is not "build a new quantum-gravity measure" but "identify the seam boundary with the $(E_8)_1$ net" — the open problem is imported into existing RCFT rigor.

The coefficient $k = \frac{c_3}{2}$ is internally forced ([`verification/v73_k_c3_half.py`]). The *dimensionless* coefficient statement splits into two cutoff-independent pieces, neither a free normalisation:

$$k = \frac{c_3}{2} = \underbrace{\frac{1}{2}}_{\text{variational}} \times \underbrace{\frac{1}{|\mathbb{Z}_2| \cdot 2\pi\,\chi(S^2)}}_{\text{Gauss–Bonnet topology}},$$

where *(a)* the factor $\frac{1}{2}$ is the universal action↔EOM factor ($16\pi G$ action vs $8\pi G$ field equation, forced by $\delta(\sqrt{g}R) \to G_{\mu\nu}$); and *(b)* $c_3 = 1/(|\mathbb{Z}_2|\cdot 2\pi\chi(S^2)) = 1/(2\cdot 4\pi) = 1/(8\pi)$ is the one-sided Gauss–Bonnet of the seam normal slice S^2 ($\chi(S^2) = 2$, hence $\int K = 4\pi$ *topological*, cutoff-independent). With this k, Fursaev–Solodukhin gives $S = 4\pi k A = 2\pi c_3 A = \frac{1}{4}A$ exactly. **So Alessandro's target is met at the level it can be:** the coefficient $k = \frac{c_3}{2}$ is forced by topology × variation; the *only* UV-sensitive piece left is the absolute 4D Newton scale (the $\Lambda^2 f_2$ prefactor of the carrier-Dirac a_2, v68) — the one irreducible *dimensionful* anchor, exactly where it should sit.

The necessary condition is met (`[verification/v59_area_law_evidence.py]`). A reflection-positive Gaussian (Calderón-type) kernel *generically* produces an area law (Srednicki 1993; Bombelli–Sorkin): for a harmonic chain the ground-state block entropy *saturates* when gapped (an area law) and grows as $\frac{c}{3}\log L$ when gapless. Crucially the physical sector is gapped — by the *same* gap $\Delta = 6\log\frac{3}{2} > 0$ that makes the attractor unique (§4) — so *one* spectral gap delivers both the unique attractor *and* the area-law form. This is the structural necessary condition for the $S = A/(4G_\Sigma)$ step; the $c_3 = \frac{1}{8\pi}$ *coefficient* still requires the specific seam kernel (the open part).

The three theses, honestly graded

Weak **[I]** (established).
: $c_3 = \frac{1}{8\pi}$ is a natural horizon-readout factor in TFPT (Gauss–Bonnet origin, Hawking/de Sitter/Unruh in one seam unit).

Medium **[P]** (very plausible).
: Black holes are local realisations of seam thermality; de Sitter is the global one.

Strong **[A]** (open, right direction).
: The seam is a holographic horizon in the mathematical sense — pending the Seam–Horizon Theorem above. *Not* more numerical matches; one analytic area-law.

9 Consequences, and the whole story [P]

Interpretive section [P] — consequences *if* the seam is a horizon

The following is the physical picture the structural facts assemble into. It is offered as a coherent reading, not a proof; the [I]/[L] core (§1–§5) does not depend on it.

Seven consequences, if the picture holds:

1. c_3 **stops being an axiom.** It is forced by horizon thermodynamics (Jacobson): P1 becomes a theorem, and the theory rests effectively on one statement — "the boundary is a horizon".
2. **The Standard Model is holographic boundary data.** One boundary transport ($\lambda_2 = (2/3)^6$) recovers information across the horizon *and* builds the flavor hierarchy (§2).
3. **Gravity is emergent/thermodynamic.** The seam's geometry channel; one Lorentzian cone, hence $v_{\mathrm{GW}} = c$ with no native dispersion.
4. **The cosmological-constant problem dissolves.** $\Lambda \sim e^{-2\alpha^{-1}} \sim 10^{-122}$ is the EM fixed point, not a fine-tuning ($122.948 = 119.028 + 3.920$, §5). **And G_N is no longer *independent*:** fixing the physical Λ branch pins $\bar{M}_{\mathrm{Pl}}$ relative to Λ (the alternative branch is mis-scaled by 661.47), so Newton's constant is a metrology readout once the physical branch and the one dimensionful induced-gravity anchor are fixed — not a free input.
5. **Information is never lost.** Unitary boundary recovery (Page rate $(2/3)^6$) resolves the information paradox by construction.

6. **The universe is cyclic and parameter-free.** The gapped transport contracts any state to the *unique* attractor (§4); the entropy reset *is* the spectral gap (Tolman's objection answered).
7. **It is falsifiable** (§7): $v_{\mathrm{GW}} \neq c$, a fourth generation, photon dispersion, horizon information loss, or birefringence $\neq 0.2424°$ would break it.

The whole story [P]

A universe ends in gravitational collapse → a horizon. By unitarity its information is not lost but encoded on the horizon (the "rand"). *That horizon is the local gravitational realisation of the TFPT seam*, carrying $c_3 = \frac{1}{8\pi}$ (the universal horizon temperature = the Einstein/Jacobson coefficient). The boundary data — the four μ_4 punctures → A_3 → N_{fam}=3 families, the D_5 carrier, the anchor $(1, 1, 2)$ — *are* the compiler. The gapped boundary transport contracts the incoming state to the unique rank-1 attractor (the "law"), discarding microstate entropy — a low-entropy reset. That law seeds the next universe with the *same* constants, because they are the attractor's fixed point; and the hull E_8 carries a built-in order-30 cyclic rotation ($30 = |\mathbb{Z}_2| N_{\mathrm{fam}} g_{\mathrm{car}}$). Repeat without end. The constants are parameter-free because they are the *self-reproducing fixed point of the cosmic cycle* — the Möbius self-consistency, made precise as Perron–Frobenius on the boundary operator.

References for the reframed standard results: Jacobson, *Thermodynamics of Spacetime* (1995); Hod (1998) and Motl (2003) for the asymptotic Schwarzschild quasinormal ln 3; Page (1993) for the recovery curve; Penrose (CCC) for the conformal far-future bounce.

10 A closure program (next phase) [A]

Following Alessandro's framing, the next phase is not "more readouts" but a TOE-*closure* program around one master principle.

Master principle: finite causal-boundary fixed-point self-consistency [P]/[A]

Physical law is the unique stable fixed point of admissible information transport across a finite causal horizon (the seam). Admissibility = unitary boundary transport + finite entropy + anomaly-free chiral matter + even-unimodular lattice closure + Hawking–Einstein 8π compatibility + primitive μ_4 seam transport + Perron–Frobenius selection of a unique stable direction. Target chain:

$$\text{causal horizon} \to \text{finite seam transport} \to \mu_4 \text{ closure} \to E_8{=}(D_5{\oplus}A_3){+}\mu_4$$
$$\to (g_{\mathrm{car}}, N_{\mathrm{fam}}){=}(5, 3) \to \text{SM} \to \text{gravity/cosmo} \to \text{predictions.}$$

This would upgrade TFPT from "compiler → readouts" to "horizon principle → compiler → universal consequences".

Two decisive theorems, and their current status.

1. *Seam = finite admissible local causal horizon* — **open** [A] (the Seam–Horizon Theorem, §8); if it closes, $c_3 = \frac{1}{8\pi}$ is physically forced. The area-law necessary condition is already met (§4, [`verification/v59_area_law_evidence.py`]); the open part is the c_3 coefficient from the seam-determinant replica.
2. *Uniqueness of the μ_4 lattice closure* — **already established** [L]: A_n has discriminant $\mathbb{Z}_{n+1}$, so μ_4 forces A_3; the rank/half-spinor conditions force D_5; hence $E_8 = (D_5{\oplus}A_3) + \mu_4$ with $(g_{\mathrm{car}}, N_{\mathrm{fam}}) = (5, 3)$ is the unique familyful simply-laced μ_4 closure ([`verification/v6_bootstrap.py`] [`verification/v15_bootstrap_classification.py`]). So three families and the $SO(10)$-type carrier are *structural consequences*, not empirical choices.

Remaining named closure targets: the D_4-equivariant Q-geometry from $\mathbb{P}^1 \setminus \mu_4$ (*now advanced [P]→[L]*: the Q-spectra and the Σ-split are derived from the $D_4 = \mathbb{Z}_4 \rtimes \mathbb{Z}_2$ structure, [verification/v69_d4_q_geometry.py]; residual = the integer lift); the functorial R-bridge ($\Lambda^2(5)$, $\Lambda^2 F$, $C_{ud}(\rho^*)$); the full covariant metric-sector equation; the derivation or explicit demotion of $N_\star$; and a prediction layer with numerical failure thresholds. The distance is now concentrated, not diffuse.

$N_\star$ — explicit demotion (Alessandro #5). The inflationary e-fold number $N_\star$ is *not* a compiler output: it is a physical reheating/observational input, marker [P], with the standard range $N_\star \in [50, 60]$ set by the reheating history. It enters only the inflationary read-offs ($n_s = 1 - 2/N_\star$, $r = 12/N_\star^2$) and *nothing* in the $[I]/[L]$ core. We state it as an input, not a derived rung; a future derivation would have to come from a reheating model, not from the compiler. **Prediction layer (Alessandro #6)** with explicit failure thresholds is now machine-recorded ([verification/v65_falsification_layer.py]).

The Seam-Engineering Index and the possibility map [I] (index) / [A] (classes B,C) ([verification/v63_seam_engineering_index.py])

TFPT is a *boundary* machine, not a particle machine: the levers are c_3 (thermal), λ_2=$(2/3)^6$ (information) and G_{metric} (geometry). The IR-stability margin has an *exact* closed form bound to the E_8 data ($\|V_{\text{metric}}\| \le 248 c_3^2 = \frac{31}{8\pi^2}$, 248= $\dim E_8$, 31=1+$h^\vee(E_8)$, the $c(E_8)$=248/31 denominator):

$$\boxed{\Xi = \frac{2\|V_{\text{metric}}\|}{\Delta} = \frac{31}{24\pi^2 \log(3/2)} \approx 0.323}, \qquad \Delta_{\text{eff}} = \Delta - 2\|V\| \approx 1.648 > 0.$$

$\Xi < 1$ is the gap-dominance (IR-stability) condition. *Honest: Ξ is a stability* ***diagnostic*** *— a parameter of the theory — not a lab control knob.* The "breakthrough" ideas then type into four honest classes:

A — allowed & near [N].
metrology/signatures: birefringence 0.2424°, BH-echo $\lesssim (2/3)^6$, v_{GW}=c, the axion window.

B — allowed as reconstruction [A].
a holographically *larger state space* (small boundary, reconstructed bulk) — gated on the open Seam–Horizon area-law.

C — only after G_{metric} [A].
topological seam shortcuts / nontrivial boundary collars — gated on the ambient metric measure.

D — currently forbidden.
local FTL, native photon dispersion, $v_{\text{GW}} \neq c$, closed timelike curves, free energy — each would *break* a core claim.

The serious next theorem is *Causal Boundary Engineering*: RP+OS+gap ⇒ no CTCs, then classify which boundary gluings G_{metric} permits — separating "shortcut" from "FTL".

Causal Boundary Engineering: a *conditional no-go* for seam shortcuts [I] (core) / [P] (chain) ([verification/v64_causal_boundary_nogo.py])

A proof attempt for the Tier-C "topological seam shortcut" (a traversable connection of causally separated regions by a *shorter* global path) turns into a no-go under the theory's own conditions:

$$\underbrace{\text{RP} + \text{OS}}_{\text{unitary causal QFT, } H\geq 0,\ \Delta>0} \quad\Rightarrow\quad \underbrace{\text{ANEC}}_{\text{Faulkner; Hartman–Kundu–Tajdini}}$$
$$\Rightarrow\ \underbrace{\text{Topological Censorship}}_{\text{Friedman–Schleich–Witt 1993}}\ \Rightarrow\ \text{no traversable shortcut,}$$

and the gapped positive H has *no periodic orbit* (verified: the transport $T^n \neq \mathbb{1}$ for all $n > 0$, $T^n \to$ rank-1 projector), the discrete analogue of *no closed timelike curve*. **Verdict:** a traversable shortcut is *forbidden* unless macroscopic ANEC (hence RP/unitarity) is broken — which would itself falsify the foundation. The only survivor is the *non-traversable* ER=EPR bridge (the $\mathbb{Z}_2$ seam): entanglement geometry, not a signal channel. Honest loophole: the quantum (Gao–Jafferis–Wall 2017) traversable wormhole exists but is *longer* than the ambient path (no FTL), i.e. again ER=EPR. So Tier C is not "open pending G_{metric}" but "forbidden unless the theory's positivity breaks".

11 Honest status

claim	status	basis
$(5,3)$ generates the integer skeleton; Pythagorean Δ_Y split	[I]	[`verification/v53_compiler_core.py`]
the 8 in c_3 is triply forced (geo = lattice = gravity)	[I]	[`verification/v54_seam_horizon_keystones.py`]
one boundary transport for flavor and horizon ($\lambda_2{=}(2/3)^6$)	[I]	[`verification/v54_seam_horizon_keystones.py`]
E_8 has an order-30 cyclic Coxeter element; $8 = \varphi(30)$	[L]	[`verification/v55_coxeter_cycle.py`]
one α^{-1} sets EM, S_{dS} and Λ ($S_{dS}\rho_\Lambda{=}32\pi^4$)	[I]/[P]	[`verification/v55_coxeter_cycle.py`]
gapped transport $\Rightarrow$ unique attractor at rate $(2/3)^6$	[I]/[L]	[`verification/v56_unique_attractor.py`]
horizon cross-links (Jacobson/Einstein $8\pi{=}1/c_3$, Hod $\ln 3$, $1920{=}\lvert W(D_5)\rvert$)	[I]/[P]	[`verification/v57_horizon_crosslinks.py`]
cyclic self-reproduction (collapse→seam→cycle)	[P]	interpretation, §6–9

One-line summary

The structural core is exact: the integer skeleton, the seam constant and a built-in order-30 cycle all flow from one boundary pair, and a gapped boundary transport selects a *unique* fixed point — so TFPT has no free fundamental number (only π is primitive). The *cyclic self-reproduction* that makes this a physical "why" is a falsifiable interpretation [P], consistent with every exact fact above but not derived from them.

Appendix: computational verification

Every [I]/[L] statement here is machine-checked in the TFPT suite and re-derived independently in Wolfram:

script	content
`v53_compiler_core.py`	$(5, 3)$ skeleton, Pythagorean Δ_Y split, anchor char-poly uniqueness
`v54_seam_horizon_keystones.py`	triply-forced 8; shared transport $\lambda_2{=}(2/3)^6$; de Sitter constants
`v55_coxeter_cycle.py`	computed E_8 Coxeter element (order 30), exponents = totatives, $S_{dS}\rho_\Lambda{=}32\pi^4$
`v56_unique_attractor.py`	gapped transport ⇒ unique attractor, rate $(2/3)^6$; Coxeter planes; rank-1 reset

Run `python verification/run_all.py` (all pass) and, for the independent second path, `wolframscript -file verification/wolfram/tfpt_readouts.wl` (all pass). Figures are produced by `verification/make_figures.py`.